Summer Work Packet for MPH Math Classes

Students going into
AP Calculus AB
Sept. 2019

Name: ________________________

This packet is designed to help students stay current with their math skills.

Each math class expects a certain level of number sense, algebra sense and graph sense in order to be successful in the course.

These problems need to be completed on a separate sheet of paper (unless space has been provided) and turned in for a grade by September 6th. Be sure to show all work. If you have any questions, please email me at dmeehan@mphschool.org.

****

The TI 84+ calculator is good for use in AP Calculus. It does everything you are allowed to use it for on the AP Exam.

****

AP CALCULUS AB

Please read the following book, LITTLE BOOK OF BIG IDEAS: Pre-calculus The Power of Functions, by Lin McMullin and do the problems indicated. (See me for the book if you don't have a copy already.) Be neat and organized. Be sure to show your work and explain in complete sentences as needed. **Do each chapter on a separate page and all graphs on graph paper.**

READ:	PROBLEMS TO DO (Show all work and support with explanations in your own words.):
Introduction	
Chapter 1	5, 8, 10, 14
Chapter 2 p. 11-16 (sec. 1 & 2 only)	4, 7, 11, 14, 15
Chapter 3	1, 4, 8, 13, 16, 20, 21a-e
Chapter 4	1a, 5, 6, 8&9 (graph w/calc.)
Chapter 5	2, 7(use sin(a-b) rule), 8, 9, 10
Chapter 6	2, 3c, d, 6, 8
Appendix: READ Polynomial, rational, exponential, logarithmic, trigonometric functions, inverse trig functions Reference Angles	**DO: Attached Activities** *Brick on a Spring *Algebraic Expressions *Graphing Trig Functions *Reference Angles *Triangle Trigonometry *Solving Trig Equations *Angle Formulas *Rational Functions *Rate of Change *Flash Cards

Brick on a Spring

Name ____________________

A brick is suspended on a spring and hangs at rest. The brick is pushed up a distance of 2 cm from its resting position. The brick is released at time t = 0 and allowed to oscillate.

1. Sketch a diagram of the brick bouncing to illustrate the ideal (never ending) situation.

2. The brick reaches its resting position after one second. Create a table that shows the position of the brick versus the time for the first 10 seconds in an ideal situation. Be sure to specify where the brick will be at zero seconds.

seconds	0	1	2	3	4	5	6	7	8	9	10
cm											

3. Create a graph (on graph paper) that models the motion of the brick in terms of time. Be sure to label the graph and axes.

Use your graph to answer questions 4-9.

4. Where is the function increasing? ________________________________

Where is it decreasing? ________________________________

5. What are the maximum values of the function? _________________

Where do they occur? ______________________ Label these points on the graph.

6. Interpret the maximum values in terms of the original physical situation.

7. What would be a reasonable domain for the function? ______________________

What range would correspond to this domain? ______________________

8. Find an equation to describe this motion.

9. Would you expect the brick to oscillate forever in real life? Explain.

10. Suppose the brick is pushed up a distance of 4 cm from its resting position, and when released still reaches its resting position after one second. Sketch a diagram illustrating the ideal situation. Draw the graph (on graph paper) and write the new equation. Be sure to label the graph and axes. How does this change the graph? How does it change the equation?

seconds	0	1	2	3	4	5	6	7	8	9	10
cm											

11. Use the slope function on your calculator to find a graph of the rate of change for the brick's position function. Sketch the rate graph (on graph paper) and find its equation.

12. Instead of being compressed, suppose the spring is pulled down so that the brick is again 2 cm from its resting position, and then released. Sketch a diagram illustrating the ideal situation. If the brick now reaches its resting position after one second, draw the graph (on graph paper) and write the new equation. Be sure to label the graph and axes.

seconds	0	1	2	3	4	5	6	7	8	9	10
cm											

13. Suppose the spring is replaced with a less bouncy spring. The brick is pushed up a distance of 4 cm from its resting position, but now takes 2 seconds to reach its resting position. Sketch a diagram illustrating the ideal situation. Draw the graph (on graph paper) and write the new equation. Be sure to label the graph and axes.

seconds	0	1	2	3	4	5	6	7	8	9	10
cm											

14. Use the slope function on your calculator to find a graph of the rate of change for the brick's position function. Sketch the rate graph (on graph paper) and find its equation.

15. Compare the position and rate graphs from question 10 & 11 to those in questions 13 &14. State the changes and explain.

Algebraic expressions Name ____________________

Simplify completely. Show your work

1. $\left(\frac{x^2+x}{x^2-4} \div \frac{x^2-1}{x^2+5x+6}\right) - \frac{4}{x^2+3x-4}$

2. $\frac{4-(1-w)^{-1}}{16+7(w^2-1)^{-1}}$

3. $\frac{25d^{-7/2}j^{8/3}}{15d^{3/2}j^{-2/3}}$

4. $\frac{\sqrt{x}-\sqrt{3}}{\sqrt{x}+\sqrt{3}}$

Solve for x. Check your answers.

5. $2x^{\frac{2}{3}}+7x^{\frac{1}{3}}-15=0$

Reference Angles, Trig and Inverse Trig Functions

a) y = sin (θ) if and only if θ = $\sin^{-1}$(y).

So, $f^{-1}(x) = \sin^{-1}(x) = \theta$ (the angle)

The domain of f(θ) = sin (θ) is all REAL numbers: $\theta \in \Re$.

The range of f(θ) is $-1 \le f(\theta) \le 1$.

The **domain** of the **inverse** function is $-1 \le x \le 1$.

The **range** of the **inverse** function is **limited.**

$\frac{-\pi}{2} \le f^{-1}(x) \le \frac{\pi}{2}$ **OR** $\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$

b) y = cos (θ) if and only if θ = $\cos^{-1}$(y).

So, $f^{-1}(x) = \cos^{-1}(x) = \theta$ (the angle)

The domain of f(θ) = cos (θ) is all REAL numbers: $\theta \in \Re$.

The range of f(θ) is $-1 \le f(\theta) \le 1$.

The **domain** of the **inverse** function is $-1 \le x \le 1$.

The **range** of the **inverse** function is **limited.**

$0 \le f^{-1}(x) \le \pi$ **OR** $0 \le \theta \le \pi$

c) y = tan (θ) if and only if θ = $\tan^{-1}$(y).

So, $f^{-1}(x) = \tan^{-1}(x) = \theta$ (the angle)

The domain of f(θ) = tan (θ)

is all REAL numbers excluding $\{\ldots -3\pi/2, -\pi/2, \pi/2, 3\pi/2, 5\pi/2, \ldots\}$ or

$\theta \neq (2k+1)\,\pi/2$ where k is an integer.

The range of f(θ) is all REAL numbers.

The **domain** of the **inverse** function is all REAL numbers.

The **range** of the **inverse** function is **limited.**

$\frac{-\pi}{2} < f^{-1}(x) < \frac{\pi}{2}$ **OR** $\frac{-\pi}{2} < \theta < \frac{\pi}{2}$

Trig & Inverse Trig Functions

Name ____________________

1. Graph the following on graph paper, from [-2π, 2π]. Use 6 blocks = π for the scale on the x-axis and 2 blocks = 1 for the y-axis. Be sure to accurately plot common reference angle points (x = $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, ...). Draw all asymptotes.

a) f(x) = sin (x)

b) g(x) = cos (x)

c) h(x) = tan (x)

d) k(x) = cot (x)

e) n(x) = sec (x)

f) p(x) = csc (x)

2. Use your calculator and sketch the **inverse functions** on the <u>same</u> axes as the <u>original</u> functions (from question #1). Pay attention to the **<u>limited</u>** range of each. Draw all asymptotes.

a) y = $\sin^{-1}$ (x)

b) y = $\cos^{-1}$ (x)

c) y = $\tan^{-1}$ (x)

Inverse Trig Functions Name ____________________

3. Use the **unit circle** to give the angle measure of each trigonometric expression. Give your answer in **radian** measure using π (no calculator). Remember the quadrants (range) for which each **inverse** function is defined.

a) $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ = ______

b) $\sin^{-1}(-1)$ = ______

c) $\cos^{-1}\left(\frac{1}{2}\right)$ = ______

d) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ = ______

e) $\cos^{-1}(0)$ = ______

f) $\sin^{-1}(1)$ = ______

m) $\cos^{-1}(-1)$ = ______

n) $\sin^{-1}\left(-\frac{1}{2}\right)$ = ______

o) $\tan^{-1}(-1)$ = ______

p) $\tan^{-1}(-\sqrt{3})$ = _____

q) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ = ______

g) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ = ______

h) $\cos^{-1}(1)$ = ______

i) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ = ______

j) $\cos^{-1}\left(-\frac{1}{2}\right)$ = ______

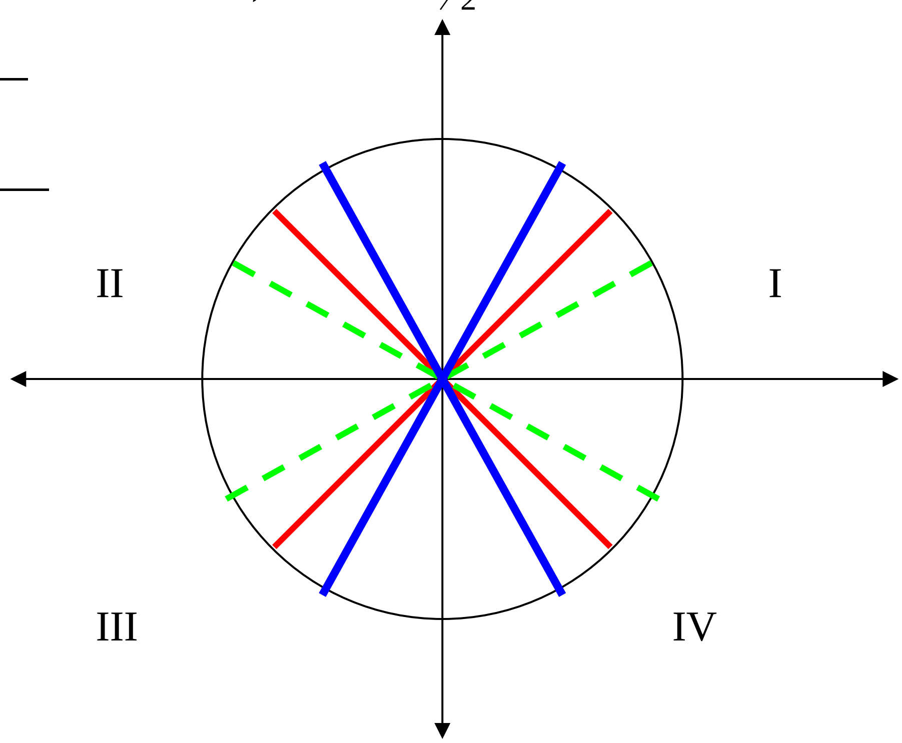
k) $\tan^{-1}(\sqrt{3})$ = ______

l) $\tan^{-1}(1)$ = ______

r) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ = ______

s) $\tan^{-1}(1)$ = ______

t) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ = ______



Inverse Trig Functions

Name ____________________

4. A. If $\cos^{-1}\left(-\frac{7}{25}\right) = \theta$ in Quadrant II, find:

B. If $\tan^{-1}\left(\sqrt{6}\right) = \alpha$ in Quadrant I, find:

(No calculator. Hint: Draw a right triangle for each.)

a) $\sin(\theta)$ = ____________________

b) $\cos(\theta)$ = ____________________

c) $\tan(\theta)$ = ____________________

d) $\cot(\theta)$ = ____________________

e) $\sec(\theta)$ = ____________________

f) $\csc(\theta)$ = ____________________

a) $\sin(\alpha)$ = ____________________

b) $\cos(\alpha)$ = ____________________

c) $\tan(\alpha)$ = ____________________

d) $\cot(\alpha)$ = ____________________

e) $\sec(\alpha)$ = ____________________

f) $\csc(\alpha)$ = ____________________

Trigonometry Name ____________________

Using the information (a)(b) = 0 if and only if a = 0 or b = 0, solve the following equation in the interval $[0, 2\pi)$. (No calculator.)

2cos(x) sin(x) – sin(x) = 0

Any point on the unit circle has the coordinates $(\cos(\theta), \sin(\theta))$, so $x = \cos(\theta)$ and $y = \sin(\theta)$. From the Pythagorean theorem, $x^2 + y^2 = 1^2$ and substituting for x and y, then **$\sin^2(\theta) + \cos^2(\theta) = 1$**.

Using the following identities, solve the following equations in the interval $[0, 2\pi)$. (No calculator.)

Pythagorean Identity: **$\sin^2(x) + \cos^2(x) = 1$** → $1 - \sin^2(x) = \cos^2(x)$ **OR** $1 - \cos^2(x) = \sin^2(x)$

Solve for x, $[0, 2\pi)$: $\sin^2(x) - \cos^2(x) = 0$

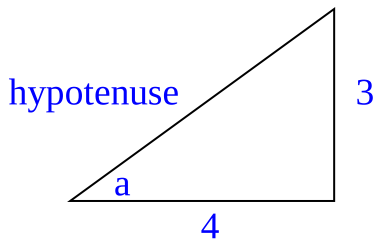
Solve for x, $[0, 2\pi)$: 2sin(x)tan(x) + tan(x) – 2 sin(x) – 1 = 0 (factor by grouping)

Trigonometry Name ____________________

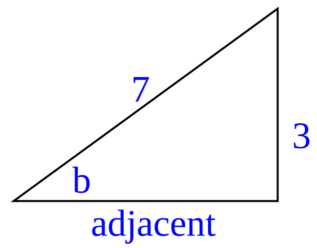
Use the formulas given on the next page to calculate the value of the given expressions on the following pages **exactly** (no calculator). Follow the examples.

GIVEN: tan(a) = ¾ and csc(b) = 7/3, in Quadrant I, find cos(a – b), tan(2b) & cos(½a).

Using a right triangle (or the Pythagorean Identity), find the values of the other trig functions.



$hyp^2 = 3^2 + 4^2$
$hyp = 5$



$3^2 + adj^2 = 7^2$
$adj = 2\sqrt{10}$

From the first triangle, $\sin(a) = \frac{3}{5}$ and $\cos(a) = \frac{4}{5}$ and $\tan(a) = \frac{3}{4}$.

From the second triangle, $\sin(b) = \frac{3}{7}$ and $\cos(b) = \frac{2\sqrt{10}}{7}$ and $\tan(b) = \frac{3\sqrt{10}}{20}$.

FIND: cos(a – b), using the formula, cos(a – b) = cos(a)cos(b) + sin(a) sin(b).

Substitute into the formula and simplify.

$$\begin{aligned}\cos(a - b) &= \cos(a)\cos(b) + \sin(a)\sin(b) \\ &= (4/5)(2\sqrt{10}/7) + (3/5)(3/7) \\ &= (8\sqrt{10} + 9)/35\end{aligned}$$

FIND: tan (2a), using the formula, $\tan(2a) = \dfrac{2\tan(a)}{1 - \tan^2(a)}$.

$$\tan(2a) = \frac{2\tan(a)}{1 - \tan^2(a)}$$

$$= \frac{2(3/4)}{1 - (3/4)^2} = \frac{24}{7}$$

FIND: cos(½b), using the formula $\cos(\tfrac{1}{2}b) = \pm\sqrt{\dfrac{1 + \cos(b)}{2}}$.

$\cos(\tfrac{1}{2}b) = \sqrt{\dfrac{1 + 2\sqrt{10}/7}{2}} = \sqrt{\dfrac{7 + 2\sqrt{10}}{14}}$ (Okay to leave in this form.)

Trigonometry Name ____________________

FORMULAS for sum & difference of angles, double angle and half-angle

$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

$\sin(a - b) = \sin(a)\cos(b) - \cos(a)\sin(b)$

$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

$\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b)$

$\tan(a + b) = \dfrac{\tan(a) + \tan(b)}{1 - \tan(a)\tan(b)}$

$\tan(a - b) = \dfrac{\tan(a) - \tan(b)}{1 + \tan(a)\tan(b)}$

$\sin(2a) = 2\sin(a)\cos(a)$

$\cos(2a) = \cos^2(a) - \sin^2(a)$

$\tan(2a) = \dfrac{2\tan(a)}{1 - \tan^2(a)}$

$\sin(½a) = \pm\sqrt{\dfrac{1 - \cos(a)}{2}}$

$\cos(½a) = \pm\sqrt{\dfrac{1 + \cos(a)}{2}}$

$\tan(½a) = \pm\sqrt{\dfrac{1 - \cos(a)}{1 + \cos(a)}}$

(The sign is determined by the quadrant.)

OR $\sin^2(a) = \dfrac{1 - \cos(2a)}{2}$ $\cos^2(a) = \dfrac{1 + \cos(2a)}{2}$ (Note: These are still double angle formulas.)

The ones in red are used most often in AP Calculus.

http://www.themathpage.com/atrig/trigonometric-identities.htm#double
Use this link to find the proof and explanation of the trigonometric identities above.

GIVEN: $\tan(a) = \frac{2}{5}$ in Quadrant III and $\cos(b) = -\frac{4}{9}$ in Quadrant II
Set up the right triangles for $\angle a$ and $\angle b$. Find the lengths of the missing side.

Trigonometry Name ____________________

Using the triangles from the previous page find:

1. $\sin(a - b)$

5. $\sin(2a)$

2. $\cos(a + b)$

6. $\cos(\frac{1}{2}b)$

3. $\tan(a - b)$

7. $\sin^2(c)$, if $\cos(2c) = \frac{1}{3}$

4. $\cos(2b)$

8. $\cos^2(c)$, if $\cos(2c) = \frac{1}{3}$

Rational Functions

Name ________________

GIVEN: $p(x) = x^2 - 1$ and $q(x) = x^3 + 1$

1. Graph $W(x) = \frac{p(x)}{q(x)}$ on graph paper. (Calculator window: Z4 to start)

2. Graph $Z(x) = \frac{q(x)}{p(x)}$ on graph paper. (Calculator window: Z4 to start)

3. Adjust your window as necessary. Check the table values. For each function, find the domain, range, end behavior model and end behavior asymptote, vertical asymptote(s), hole(s), local/absolute maximum(s) and minimum(s) and real root(s).

W(x)	Z(x)
Domain:	Domain:
Range:	Range:
VA:	VA:
Holes:	Holes:
EBM:	EBM:
EBA:	EBA:
Max:	Max:
Min:	Min:
Real Roots:	Real Roots:

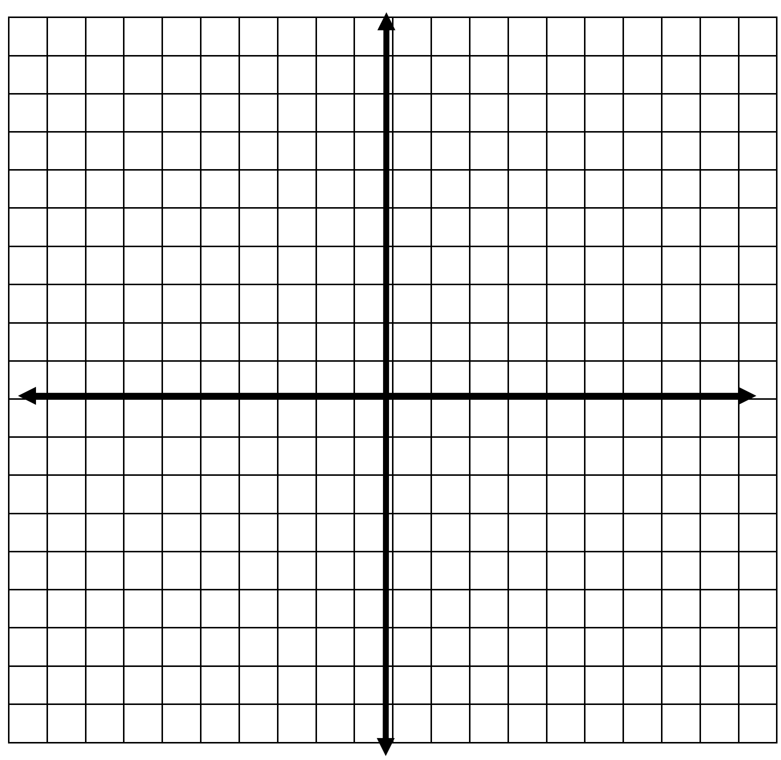
Rate of Change

Name ______________________

1. **GIVEN: $P(x) = 2x^3 - 3x^2 + 4x - 5$**
   Using the difference quotient and h = 0, find the equation of the
   A) velocity function, and
   B) acceleration function.

2. **Using the slope function and the table on your calculator, find the equation of the velocity function. Sketch the graphs of both functions below.**

   GIVEN: $P(x) = \ln(x)$



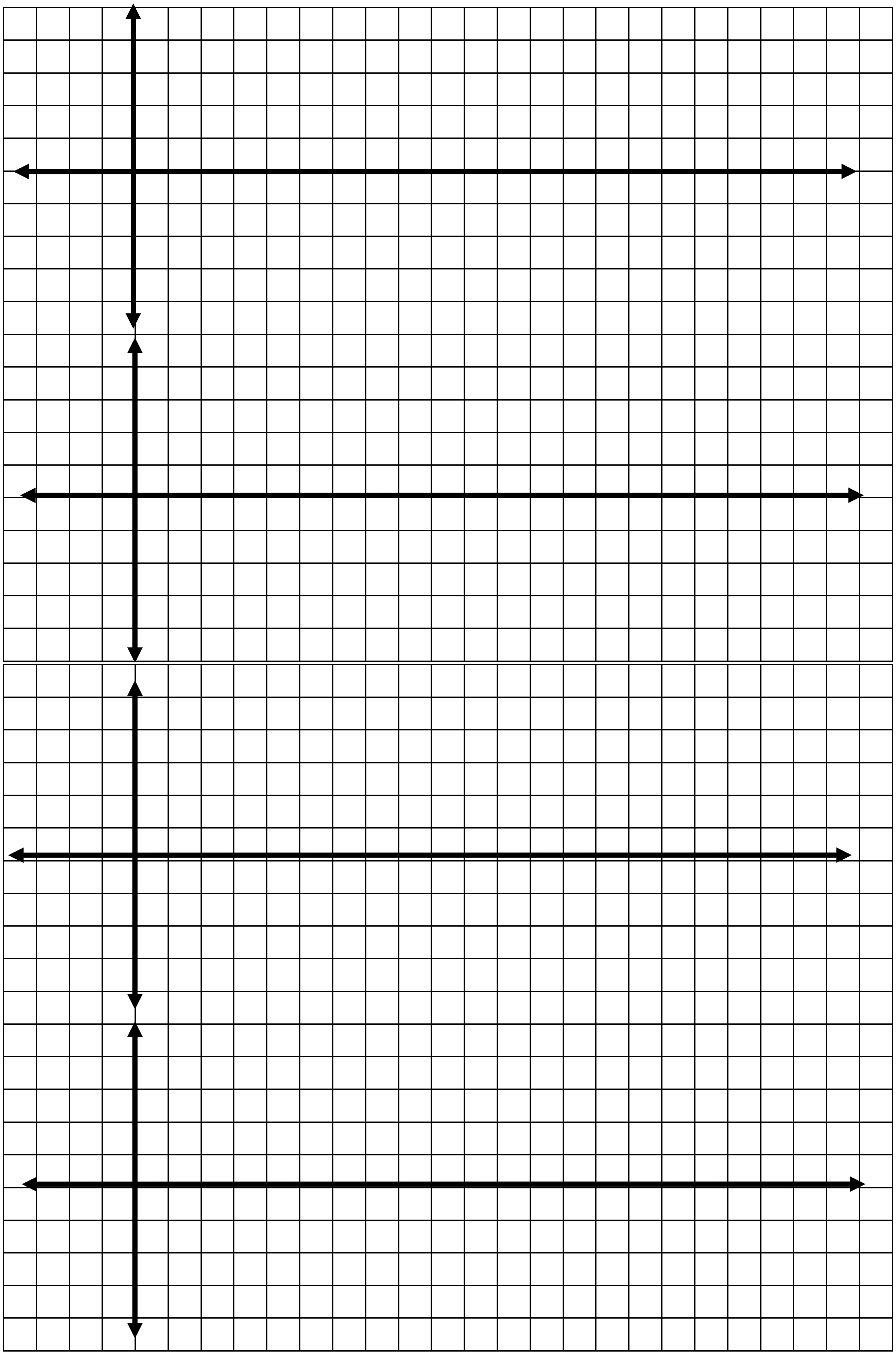
3. **GIVEN: P(x) = sin(x) Using the slope function on your calculator, find the equation of the A) velocity function,**

B) acceleration function, and

C) jerk function.

D) Sketch the graphs of all four functions below from $[0, 2\pi]$**.**

Use 6 blocks = π for the scale on the x-axis and 2 blocks = 1 for the y-axis.



P(x) = sin(x)

V(x) = ______________

A(x) = ______________

J(x) = ______________

Parent Functions and Transformations Name ____________________

Create a deck of cards for matching. You will need to know these graphs.

A) Write the parent function and state its domain and range on a card.
B) Draw its graph on a second card.
C) Write the transformation described on a third card.
D) Draw the graph for the transformation described on a fourth card.
E) Write the equation for the transformation described on a fifth card.

You will need 70 cards total. Use a different color for each group.

Parent Functions	**Transformations**
1. $f(x) = x$	Vertical shrink of ⅓.
2. $f(x) = \lvert x \rvert$	Horizontal shift right 2, vertical stretch of 3
3. $f(x) = [\![x]\!]$	Vertical shift up 1
4. $f(x) = x^2$	Vertical stretch of 2
5. $f(x) = x^3$	Horizontal shift left 3
6. $f(x) = \frac{1}{x}$	Horizontal shift left 2, vertical shift down 1
7. $f(x) = \sqrt{x}$	Horizontal shift left 3
8. $f(x) = e^x$	Vertical shrink of ½
9. $f(x) = \frac{1}{2}^x$	Vertical stretch of 4, vertical shift up 1
10. $f(x) = \ln(x)$	Horizontal shift left 3
11. $f(x) = \sin(x)$ from $[0, 2\pi]$	Horizontal shrink of 3
12. $f(x) = \cos(x)$ from $[0, 2\pi]$	Horizontal stretch of 2
13. $f(x) = \tan(x)$ from $[0, 2\pi]$	Reflection over the x axis
14. $x^2 + y^2 = 1$	Horizontal shift right 1, vertical shift down 1