Summer Work Packet for MPH Math Classes

Students going into AP Calculus AB Sept. 2017

Name:_______________________

This packet is designed to help students stay current with their math skills.

Each math class expects a certain level of number sense, algebra sense and graph sense in order to be successful in the course.

These problems need to be completed on a separate sheet of paper (unless space has been provided) and turned in for a grade by September 8th. Be sure to show all work. If you have any questions, please email me at dmeehan@mphschool.org.

The TI 84+ calculator is good for use in AP Calculus. It does everything you are allowed to use it for on the AP Exam.

AP CALCULUS AB

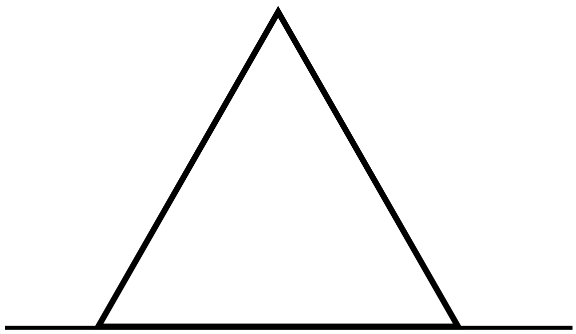
Please read the following book, LITTLE BOOK OF BIG IDEAS: Pre-calculus The Power of Functions, by Lin McMullin and do the problems indicated. (See me for the book if you don't have a copy already.) Be neat and organized. Be sure to show your work and explain in complete sentences as needed. **Do each chapter on a separate page and all graphs on graph paper.**

READ:	PROBLEMS TO DO (Show all work and support with explanations in your own words.):
Introduction	
Chapter 1	5, 8, 10, 14
Chapter 2 p. 11-16 (sec. 1 & 2 only)	4, 7, 11, 14, 15
Chapter 3	1, 4, 8, 13, 16, 20, 21a-e
Chapter 4	1a, 5, 6, 8&9 (graph w/calc.)
Chapter 5	2, 7, 8, 9, 10
Chapter 6	3, 6, 8
Appendix: READ Polynomial, rational, exponential, logarithmic, trigonometric functions	DO: Attached Activities *Amusement Park *Reference Angles *Inverse Trig Functions *Trigonometry *Rational Functions *Rate of Change *Flash Cards

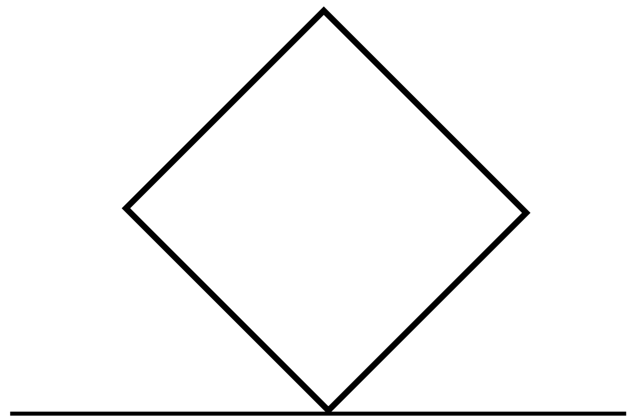
Amusement Park Rides Name ____________________

Roller Coaster Rides

Imagine a roller coaster ride in the MathLand Amusement Park! There are two very strange roller coaster rides. Instead of making a loop, one roller coaster ride goes along a triangle track and the other on a square track, tilted at a 45° angle.



Cart



Cart

Part 1

The triangle is an equilateral triangle, with each side equal to 60 feet. The square is also 60 feet on each side. The roller coaster car travels at 20 feet per second along the track. The car makes three complete loops before continuing along the track.

Your job is to create a **position versus time** and a **velocity versus time** graph for **vertical height** above the ground for each of these rides. Be sure to indicate in the diagram which direction the cart is traveling when it starts.

Ferris Wheel Ride

Imagine sitting on a Ferris wheel as it is turning. We can think of this motion as circular since our body follows the circular path of the Ferris wheel. However, we can also think of this motion as a combination of vertical motion and horizontal motion. Some of the most useful application of the trigonometric functions lie in their ability to separate circular motion into its vertical and horizontal components.

Suppose a Ferris wheel with an 80 foot diameter makes one revolution every 24 seconds in a counterclockwise direction. The Ferris wheel is built so that the lowest seat on the wheel is 10 feet off the ground. This particular Ferris wheel has a boarding platform which is located at a height that is exactly level with the center (or hub) of the Ferris wheel. You take your seat level with the hub as the ride begins.

Draw a diagram of the Ferris wheel, with its lowest cart 10 feet off the ground and the boarding platform for a cart level with the center of the wheel.

Part 2

1. What is your height above the hub after 3 seconds (round you answer to the nearest thousandth)? How long does it take for you to reach the highest point? How high above the ground is that point?

2. Do you rise more in the first three seconds or in the next three seconds? Explain how you found your answer.

3. What is your height above the hub after 9 seconds? After 12 seconds? After 15 seconds? After 36 seconds?

4. Create a table that represents the relationship between time (t = 0, t = 3, t = 6, etc.) and height above the hub for at least two revolutions of the Ferris wheel. Explain how you determined these values.

5. Create a graph **(on graph paper)** and an equation that shows height above the hub as a function of time. Check the equation with your table values.

6. If the ride lasts six minutes, what is the domain and range of your function in this context?

Part 3

7. How would your function change for each of the following situations? Be sure to show both **<u>an equation and a graph</u> (on graph paper)** for each situation. Describe how the original function was transformed.

   a. How would your function change if you wanted to know the height above the ground rather than above the hub?

   b. How would your function change if the Ferris wheel rotated three times as fast? Half as fast?

   c. How would your function change if the diameter of the Ferris wheel were 73 feet?

d. How would your function change if you board at the bottom of the Ferris wheel? (You still measure your height above the hub.) **Include a diagram of the Ferris wheel with your graph (on graph paper) and equation.**

e. How would your function change if the boarding platform is moved to the bottom of the Ferris wheel? Find your height above this new boarding platform as a function of time. **Include a diagram of the Ferris wheel with your graph (on graph paper) and equation**.

Part 4

8. Use your original equation that shows the height above the hub as a function of time. Carefully sketch the graph of your function for two complete revolutions of the Ferris wheel.

   a. Use the slope function on your calculator to find the average rate at which the height is changing for two complete revolutions of the Ferris wheel. Carefully sketch the **graph** of the **rate** at which the height is changing as a function of time for two revolutions. Find an **equation** that approximates this graph.

   b. Is the velocity ever zero? If so, when does this occur? What is the height of the wheel when the velocity is zero?

   c. If the Ferris wheel rotated twice as fast, how would your position graph change? What would be the corresponding change in the velocity graph? Carefully sketch the new position **graph** and the **rate** of change graph. Find **equations** that approximate the graphs.

Reference Angles and Inverse Trig Functions

a) y = sin (θ) if and only if θ = $\sin^{-1}$(y).

So, $f^{-1}(x) = \sin^{-1}(x) = \theta$ (the angle)

The domain of f(θ)= sin (θ)is all REAL numbers: $\theta \in \Re$.

The range of f(θ) is $-1 \le f(\theta) \le 1$.

The **domain** of the **inverse** function is $-1 \le x \le 1$.

The **range** of the **inverse** function is **limited.**

$\frac{-\pi}{2} \le f^{-1}(x) \le \frac{\pi}{2}$ **OR** $\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$

b) y = cos (θ) if and only if θ = $\cos^{-1}$(y).

So, $f^{-1}(x) = \cos^{-1}(x) = \theta$ (the angle)

The domain of f(θ) = cos (θ) is all REAL numbers: $\theta \in \Re$.

The range of f(θ) is $-1 \le f(\theta) \le 1$.

The **domain** of the **inverse** function is $-1 \le x \le 1$.

The **range** of the **inverse** function is **limited.**

$0 \le f^{-1}(x) \le \pi$ **OR** $0 \le \theta \le \pi$

c) y = tan (θ) if and only if θ = $\tan^{-1}$(y).

So, $f^{-1}(x) = \tan^{-1}(x) = \theta$ (the angle)

The domain of f(θ) = tan (θ)

is all REAL numbers excluding $\{\ldots -3\pi/2, -\pi/2, \pi/2, 3\pi/2, 5\pi/2, \ldots\}$ or

$\theta \ne (2k+1)\,\pi/2$ where k is an integer.

The range of f(θ) is all REAL numbers.

The **domain** of the **inverse** function is all REAL numbers.

The **range** of the **inverse** function is **limited.**

$\frac{-\pi}{2} < f^{-1}(x) < \frac{\pi}{2}$ **OR** $\frac{-\pi}{2} < \theta < \frac{\pi}{2}$

Inverse Trig Functions Name ____________________

1. Graph the following on graph paper, from $[-\pi, 2\pi]$. Use 6 blocks = π for the scale on both axes.

a) $f(x) = \sin(x)$
b) $g(x) = \cos(x)$
c) $h(x) = \tan(x)$
d) $k(x) = \cot(x)$
e) $n(x) = \sec(x)$
f) $p(x) = \csc(x)$

2. Use your calculator and sketch the **inverse functions** on the same axes as the original functions. Pay attention to the **limited** range of each.

a) $y = \sin^{-1}(x)$
b) $y = \cos^{-1}(x)$
c) $y = \tan^{-1}(x)$

3. Use the **unit circle** (next page) to give the angle measure of each trigonometric expression. Give your answer in **radian** measure using π (no calculator). Remember the quadrants for which each inverse function is defined.

a) $\tan^{-1}\left(-\sqrt{3}/3\right)$ = ______

b) $\sin^{-1}(-1)$ = ______

c) $\cos^{-1}\left(1/2\right)$ = ______

d) $\sin^{-1}\left(-\sqrt{3}/2\right)$ = ______

e) $\cos^{-1}(0)$ = ______

f) $\sin^{-1}(1)$ = ______

g) $\cos^{-1}\left(-\sqrt{2}/2\right)$ = ______

h) $\cos^{-1}(1)$ = ______

i) $\sin^{-1}\left(\sqrt{3}/2\right)$ = ______

j) $\cos^{-1}\left(-1/2\right)$ = ______

k) $\tan^{-1}\left(\sqrt{3}\right)$ = ______

l) $\tan^{-1}(1)$ = ______

m) $\cos^{-1}(-1)$ = ______

n) $\sin^{-1}(-\frac{1}{2})$ = ______

o) $\tan^{-1}(-1)$ = ______

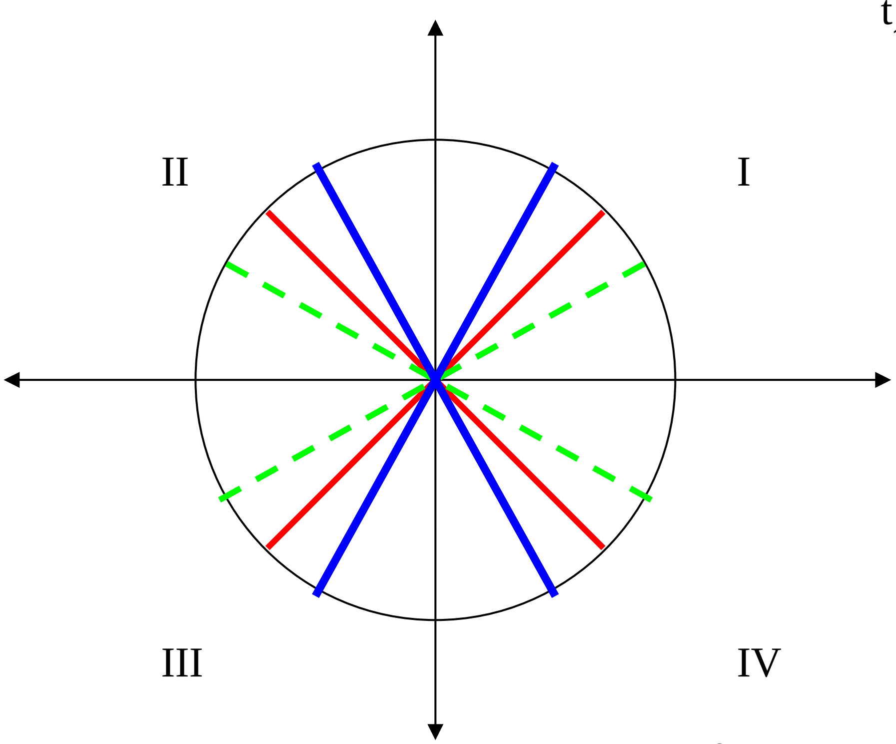
p) $\tan^{-1}(-\sqrt{3})$ = _____

q) $\cos^{-1}(\frac{\sqrt{3}}{2})$ = ______

r) $\tan^{-1}(\frac{\sqrt{3}}{3})$ = ______

s) $\tan^{-1}(1)$ = ______

t) $\cos^{-1}(-\frac{\sqrt{3}}{2})$ = ______



(No calculator - hint: Draw a right triangle for each.)

4 A. If $\cos^{-1}(\frac{5}{13}) = \theta$ find:

a) $\sin(\theta)$ = ________________________

b) $\cos(\theta)$ = ________________________

c) $\tan(\theta)$ = ________________________

d) $\cot(\theta)$ = ________________________

e) $\sec(\theta)$ = ________________________

f) $\csc(\theta)$ = ________________________

B. If $\tan^{-1}(\frac{\sqrt{3}}{3}) = \alpha$, find:

a) $\sin(\alpha)$ = ________________________

b) $\cos(\alpha)$ = ________________________

c) $\tan(\alpha)$ = ________________________

d) $\cot(\alpha)$ = ________________________

e) $\sec(\alpha)$ = ________________________

f) $\csc(\alpha)$ = ________________________

Trigonometry Name ____________________

Using the information (a)(b) = 0 if and only if a = 0 or b = 0, solve the following equation in the interval $[0, 2\pi)$. (No calculator.)

$2\sin(x)\cos(x) - \cos(x) = 0$

Using the identities, solve the following equations in the interval $[0, 2\pi)$. (No calculator.)

Pythagorean Identity: $\mathbf{\sin^2(x) + \cos^2(x) = 1}$ $1 - \sin^2(x) = \cos^2(x)$ $1 - \cos^2(x) = \sin^2(x)$

Solve for x, $[0, 2\pi)$: $\sin^2(x) - \cos^2(x) = 0$

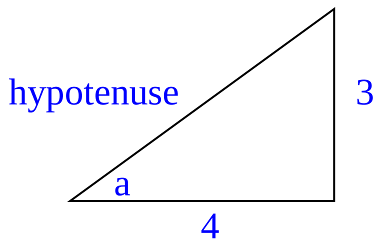
Solve for x, $[0, 2\pi)$: $2\cos(x)\tan(x) + \tan(x) - 2\cos(x) - 1 = 0$

Trigonometry Name ____________________

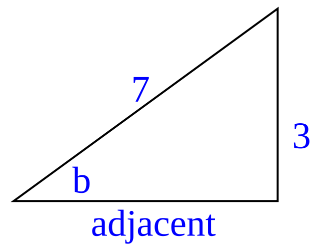
Use the formulas given on the next page to calculate the value of the given expressions on the following pages **exactly** (no calculator). Follow the examples.

GIVEN: tan (a) = ¾ and csc (b) = 7/3, in Quadrant I, find cos (a – b), tan (2b) & cos (½ a).

Using a right triangle (or the Pythagorean Identity), find the values of the other trig functions.



$hyp^2 = 3^2 + 4^2$
hyp = 5



$3^2 + adj^2 = 7^2$
adj = $2\sqrt{10}$

From the first triangle, sin(a) = $\frac{3}{5}$ and cos(a) = $\frac{4}{5}$ and tan(a) = $\frac{3}{4}$.

From the second triangle, sin(b) = $\frac{3}{7}$ and cos(b) = $\frac{2\sqrt{10}}{7}$ and tan(b) = $\frac{3\sqrt{10}}{20}$.

FIND: cos (a – b), using the formula, cos (a – b) = cos(a)cos(b) + sin(a) sin(b).

Substitute into the formula and simplify.

cos (a – b) = cos(a)cos(b) + sin(a) sin(b)
= $(4/5)(2\sqrt{10}/7) + (3/5)(3/7)$
= $(8\sqrt{10}+9)/35$

FIND: tan (2a), using the formula, $\tan(2a) = \frac{2\tan(a)}{1-\tan^2(a)}$.

$$\tan(2a) = \frac{2\tan(a)}{1-\tan^2(a)}$$

$$= \frac{2(3/4)}{1-(3/4)^2} = \frac{24}{7}$$

FIND: cos (½b), using the formula cos (½b) = $\pm\sqrt{\frac{1+\cos(b)}{2}}$.

cos (½b) = $\sqrt{\frac{1+2\sqrt{10}/7}{2}} = \sqrt{\frac{7+2\sqrt{10}}{14}}$ (Okay to leave in this form.)

Trigonometry Name ____________________

FORMULAS for sum & difference of angles, double angle and half-angle

sin(a + b) = sin(a)cos(b) + cos(a) sin(b)

sin(a – b) = sin(a)cos(b) – cos(a) sin(b)

cos(a + b) = cos(a)cos(b) – sin(a) sin(b)

cos(a – b) = cos(a)cos(b) + sin(a) sin(b)

tan(a + b) = $\frac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)}$

tan(a – b) = $\frac{\tan(a)-\tan(b)}{1+\tan(a)\tan(b)}$

sin(2a) = 2sin(a) cos(a)

cos(2a) = $\cos^2$(a) – $\sin^2$(a)

$\tan(2a) = \frac{2\tan(a)}{1-\tan^2(a)}$

sin(½a) = $\pm\sqrt{\frac{1-\cos(a)}{2}}$

cos(½a) = $\pm\sqrt{\frac{1+\cos(a)}{2}}$

tan(½a) = $\pm\sqrt{\frac{1-\cos(a)}{1+\cos(a)}}$

(The sign is determined by the quadrant.)

OR $\sin^2$(a) = $\frac{1-\cos(2a)}{2}$

$\cos^2$(a) = $\frac{1+\cos(2a)}{2}$ (Note: These are still double angle formulas.)

GIVEN: tan(a) = $\frac{2}{3}$ in Quadrant I and cos(b) = $\frac{12}{13}$ in Quadrant I

FIND:

1. cos (a + b)

2. sin (a – b)

3. tan (a – b)

4. sin (2a)

5. $\tan(2a)$

6. $\cos(2b)$

7. $\cos(\frac{1}{2}a)$

8. $\tan(\frac{1}{2}a)$

9. $\sin^2(2a)$

10. $\cos^2(2b)$

Rational Functions Name ________________

GIVEN: $p(x) = x^2 - 1$ and $q(x) = x^3 + 1$

1. Graph $W(x) = \frac{p(x)}{q(x)}$ on graph paper. Window: Z4

2. Graph $Z(x) = \frac{q(x)}{p(x)}$ on graph paper. Window: Z4

3. For *Z(x)* change the y-max to 6.2 and adjust your graph. Check the table values. For each function, find the domain, range, end behavior model and end behavior asymptote, vertical asymptote(s), hole(s), local/absolute maximum(s) and minimum(s) and real root(s).

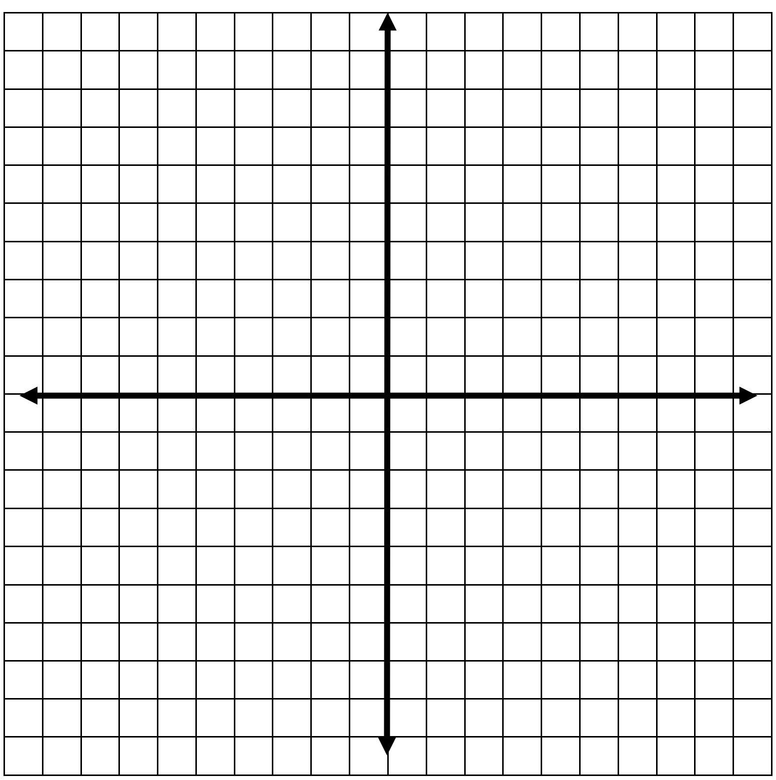
W(x)	Z(x)
Domain:	Domain:
Range:	Range:
VA:	VA:
Holes:	Holes:
EBM:	EBM:
EBA:	EBA:
Max:	Max:
Min:	Min:
Real Roots:	Real Roots:

Rate of Change

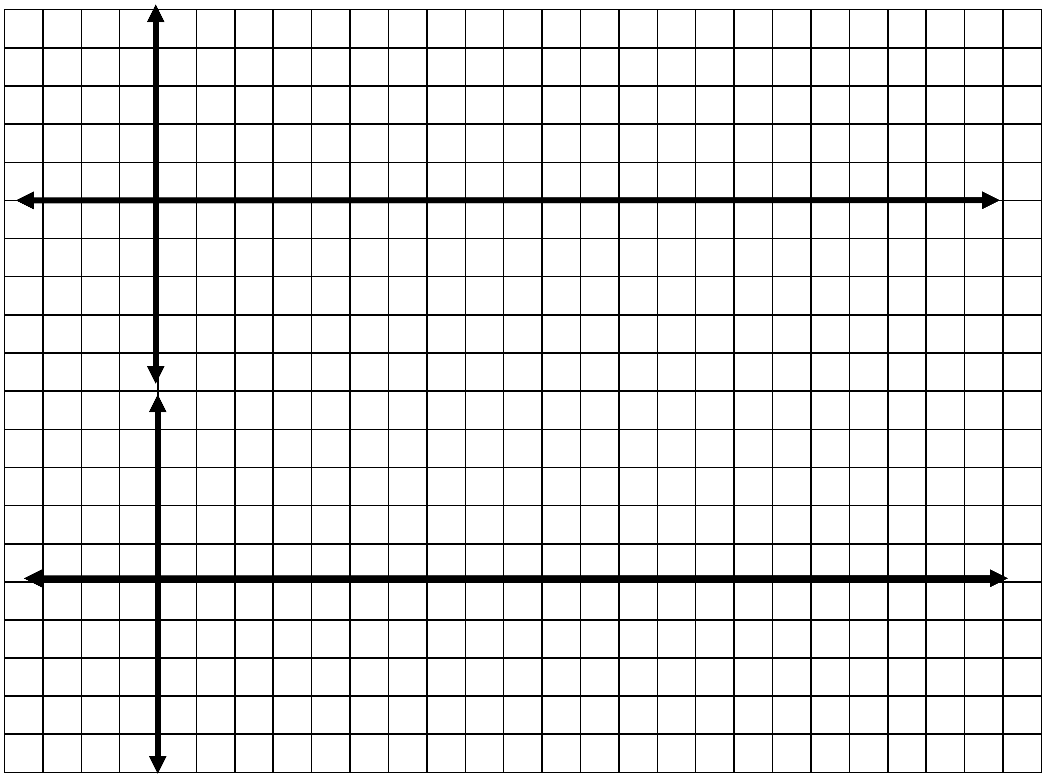
Name ______________________

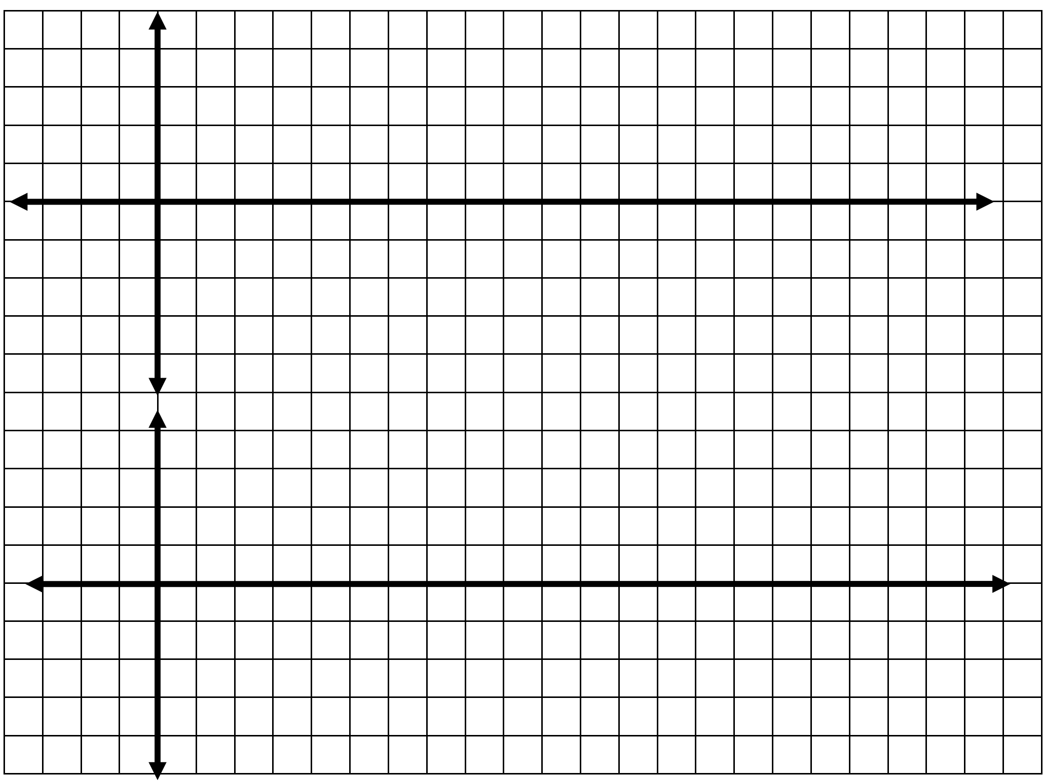
1. **Find the equation of the A) velocity function and B) acceleration function using the difference quotient and h = 0.** **GIVEN: $P(x) = 4x^3 - 3x^2 + 2x - 5$**

2. **Find the equation of the velocity function using the slope function on your calculator. Sketch the graphs of both functions below.** **GIVEN: $P(x) = \ln(x)$**



3. Find the equation of the A) velocity function, B) acceleration function and C) jerk function using the slope function on your calculator. Sketch the graphs of all four functions below from $[0, 2\pi]$**.** **GIVEN: P(x) = sin (x)**





Parent Functions and Transformations

Name ____________________

Create a deck of cards for matching:

A) the parent function,
B) its graph, and
C) its domain and range,
D) the transformation described, and
E) a graph & equation for the transformation described.

You will need 75 cards total. Use a different color for each group.

Parent Functions	Transformations
1. $f(x) = x$	Vertical stretch of 2
2. $f(x) = \|x\|$	Horizontal shift left 3
3. $f(x) = [x]$	Vertical shift down 1
4. $f(x) = x^2$	Vertical shrink of ½
5. $f(x) = x^3$	Horizontal shift right 2
6. $f(x) = \frac{1}{x}$	Horizontal shift left 2, vertical shift up 1
7. $f(x) = e^x$	Horizontal shift right 3
8. $f(x) = b^x$ if $b > 1$	Vertical stretch of 3
9. $f(x) = b^x$ if $b < 1$	Vertical shrink of ½ , vertical shift up 1
10. $f(x) = \log(x)$	Horizontal shift left 1

11. $f(x) = \ln(x)$	Horizontal shift right 1
12. $f(x) = \sin(x)$	Horizontal stretch of 2
13. $f(x) = \cos(x)$	Horizontal shrink of 2
14. $f(x) = \tan(x)$	Reflection over the x axis
15. $x^2 + y^2 = 1$	Horizontal shift right 1, vertical shift down 1