Summer Work Packet for MPH Math Classes

Students going into AP Calculus BC Sept. 2017

Name:________________________

AP CALCULUS BC

There are three main parts to this packet. Please follow each set of instructions for each part. For this packet you will need to watch the movie "Hidden Figures". It is available on amazon prime (the link is below).

https://www.amazon.com/Hidden-Figures-Taraji-P-Henson/dp/B01LTI1RHG/ref=sr_1_2?ie=UTF8&qid=1498057398&sr=8-2&keywords=hidden+figures+movie

Students will need TI-84 or TI-89 calculator for AP Calculus BC.

Good luck, have fun and please contact me or Mrs. Meehan with any concerns or problems!

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Part I: Introduction to Sequences and Sums

Instructions:

1. Go to https://www.khanacademy.org/
2. Click on "Learners, start here"
3. Under **Math by subject**, click "AP Calculus BC"
4. Scroll down until you reach the section that says "Sequences & series intro"

You will complete lessons and practice exercises from four topics *off the menu on the left hand side*. I have laid out all the topics with the videos (lessons) you are responsible for watching, followed by their corresponding practice exercises.

You are to complete the practice exercises on a loose leaf sheet of paper. This will be collected within the first week of school. Please let me know if you have any questions.

Sequences Review:

1. Videos: "Sequences Intro", "Worked Example Sequence: Explicit Formula", "Worked Example Sequence: Implicit Formula"
2. Practice: "Sequences Review"

Infinite Sequences:

1. Videos: "Convergent and Divergent Sequences" , "Worked Example: Sequence Convergent/Divergent"
2. Practice: Sequence Convergent/Divergent

Series Review:

1. Videos: "Sigma Notation for Sums", "Worked Example: Sigma Notation", "Worked Example: Sigma Notation ($n \geq 2$) "
2. Practice: "Sigma Notation intro"

Partial Sums:

1. Videos: "Partial Sums intro", "Partial Sums: formula for nth term from partial sum", "Partial Sums: term value form partial sum"
2. Practice: "Partial Sums Intro"

Part II: Hidden Figures

Instructions: Please watch the movie "Hidden Figures" and answer the following questions. This will be collected and graded.

You may access the movie through amazon using the link below:

https://www.amazon.com/Hidden-Figures-Taraji-P-Henson/dp/B01LTI1RHG/ref=sr_1_2?ie=UTF8&qid=1498057398&sr=8-2&keywords=hidden+figures+movie

1. **Characters:**

Give the first and last names of each person.
The three women who work at NASA;

______________________, ______________________, ______________________

Which of the three women is …
the engineer? ______________________
the computer scientist? ______________________
the mathematician? ______________________

The head of the whole Space Task Group: ______________________
The engineer who is "second in command" in the Space Task Group: ______________________

2. **Setting:**

Where does the movie take place? ______________________
Is the movie fiction, non-fiction, or somewhere in between the two? ______________________
Who was the U.S. president at the time?______________________
Was Martin Luther King Jr. still alive? ______________________
Give an example of how "colored" were treated differently than "whites".

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Give an example of how women were treated differently than men.

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3. **Katherine** gets an assignment to the Space Task Group.
Why does she get the assignment?
What kind of math can she do?

4. **Mary** wants to become an engineer but needs to take additional classes.
Where does she take the classes?
Who does she have to convince to let her enroll?

5. **Dorothy** sneaks a book out of the library,
What kind of book is it?
Why does she have to sneak it out?
Why does she need the book?

6. Who was the first Russian to orbit the Earth? ______________________________
How many times did he orbit the earth? ___________________________________
Who was the first American to orbit the Earth? _____________________________
How many times did he orbit the Earth? __________________________________
Who orbited first, a Russian or an American? ______________________________

7. Who got the IBM (I ________________B_________________M________________)?

How many multiplications can the IBM compute per second? ________________

8. Who figured out the landing coordinates? _________________________
Using what method? ______________________
Was this new math or old math? ________________________

9. Who knocked down the "colored ladies room" sign? ______________________
Explain why.

10. What parting gift does Katherine get right before her wedding? ____________________
What is the significance of the gift?

11. Right before John Glenn's lift off, what happens?
Who does John Glenn ask to check the coordinates?

12. Who becomes the supervisor at the end of the movie? ___________________________
What does Mrs. Mitchell call her at that time and what is the significance of this?

13. How many years were Katherine and Jim married?_______
What happened to Katherine's first husband? ________________

14. When Mary goes to night school, where is the "colored section" of the classroom?

15. State the number of children each woman has.

Katherine _____ Mary ______ Dorothy _____

16. How many times is John Glenn scheduled to orbit the Earth? _____ How long is it supposed to take? _____ How many times does he actually orbit the Earth? ____ Why did they have to bring him down early? _________

17. Where did John Glenn take off? __________________________

18. Where did John Glenn land? ____________________________

19. What does Paul bring Katherine at the end of the movie? ___________________
What is the significance of this?

PART III : INTRODUCTION TO BOUNDED FUNCTIONS

Please make sure to read this **carefully** and complete the examples. This will be collected within the first week of school.

Consider the function $\sin(x)$. We know that for all x in $\mathbb{R}$, $-1 \leq \sin x \leq 1$. Recall that this is equivalent to saying that

$$|\sin x| \leq 1 \text{ for all } x \text{ in } \mathbb{R}.$$

In this case we say that 1 is a **bound for** $\sin x$ **on** $\mathbb{R}$. Intuitively, a bound for a function is a real number M such that the absolute value of the function never exceeds M.

DEFINITION 1. Let f be a function defined on a closed interval $[a, b]$ and let M be a positive real number. We say M is a **bound** for $f(x)$ if

$$|f(x)| \leq M \text{ for all } x \text{ in } [a, b] \text{ and } M \geq 0.$$

An important point to notice is that bounds are not unique. While it is true that $|\sin x| \leq 1$ for all x in $\mathbb{R}$, it is also true that $|\sin x| \leq 2$, $|\sin x| \leq 3$, $\ldots$, $|\sin x| \leq 100$ etc. However, 1 is the smallest bound so we think of it as the "best" in some sense since anything smaller than 1 is **not** a bound for $\sin x$. Let's work out some explicit examples to see what this means.

EXAMPLE 1. Let $f(x) = \sin x$. State three bounds for $f(x)$ and determine the smallest bound for $f(x)$.

Solution. Since $|f(x)| \leq 1$ for all x in $\mathbb{R}$, then 3 bounds for $f(x)$ could be $1, 5$, and 17. But as stated above, the smallest bound for $f(x)$ is 1. This can be seen by looking at the graph of $\sin x$.

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EXAMPLE 2. Let $f(x) = 2\sin x + 1$. State three bounds for $f(x)$ and determine the "best" (smallest) bound for $f(x)$.

Now consider the function $f(x) = x$. This linear function does not have a bound on the whole real line. In other words, there is no number M such that $|x| \leq M$ for all x in $\mathbb{R}$. Graphically we understand this because $|x|$ goes to infinity for both positive and negative x.

But when we limit the domain values to a closed interval, for example $[1, 3]$, the y values are limited as well. So in general we care mostly about continuous functions defined on a closed interval because these functions can always be bounded. In practice the Extreme Value Theorem gives us a procedure to find a bound.

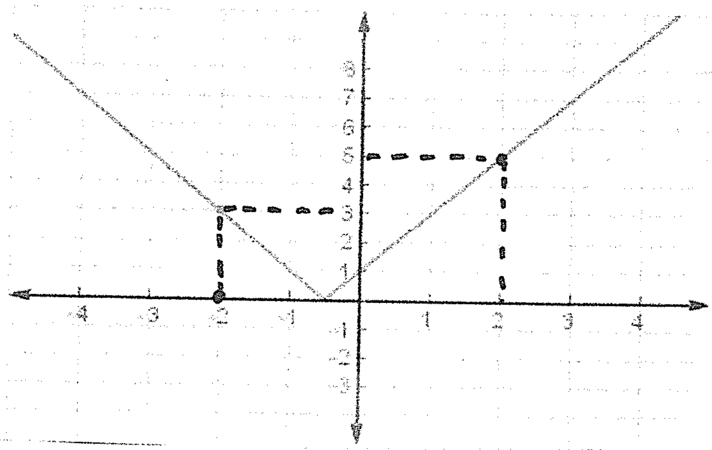
THEOREM 1 (EXTREME VALUE THEOREM). Let $f(x)$ be a continuous functions defined on a closed interval $[a, b]$. Then, $f(x)$ attains both a maximum and a minimum on $[a, b]$.

Our goal is to find bounds for the absolute value of functions. We will doing this by using the Extreme Value Theorem and the following theorem.

THEOREM 2. If $f(x)$ is a continuous function on the closed interval $[a, b]$, then $|f(x)|$ is also continuous on $[a, b]$.

Combining these two, if $f(x)$ is continuous on $[a, b]$, then $|f(x)|$ attains a maximum on $[a, b]$.

EXAMPLE 3. In this example we will be finding a bound for $f(x) = 2x + 1$ on the closed interval $[-2, 2]$. The first step is to graph $|f(x)| = |2x + 1|$.



Next find the maximum of $|f(x)|$ on the given interval. In this example, since 5 is the largest y-value attained on $[-2, 2]$, we have that

$$|f(x)| \leq 5 \text{ for all x in } [-2, 2].$$

So, $M = 5$ and we have found our bound for $f(x)$ on $[-2, 2]$.

Please complete the following examples, this will be collected. Find a bound for each of the following functions on the given intervals. Please make sure to provide graphs of $f(x)$ *and* $|f(x)|$.

1. $f(x) = e^x$ on $\left[\frac{1}{2}, 1\right]$
2. $f(x) = \dfrac{1}{x}$ on $[-3, -2]$
3. $f(x) = \ln x$ on $\left[\frac{1}{4}, 1\right]$ and $[1, 2]$.