Summer Work Packet for MPH Math Classes

Students going into
AP Calculus AB
Sept. 2018

Name: ______________________

This packet is designed to help students stay current with their math skills.

Each math class expects a certain level of number sense, algebra sense and graph sense in order to be successful in the course.

These problems need to be completed on a separate sheet of paper (unless space has been provided) and turned in for a grade by September 7th. Be sure to show all work. If you have any questions, please email me at dmeehan@mphschool.org.

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The TI 84+ calculator is good for use in AP Calculus. It does everything you are allowed to use it for on the AP Exam.

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AP CALCULUS AB

Please read the following book, LITTLE BOOK OF BIG IDEAS: Pre-calculus The Power of Functions, by Lin McMullin and do the problems indicated. (See me for the book if you don't have a copy already.) Be neat and organized. Be sure to show your work and explain in complete sentences as needed. **Do each chapter on a separate page and all graphs on graph paper.**

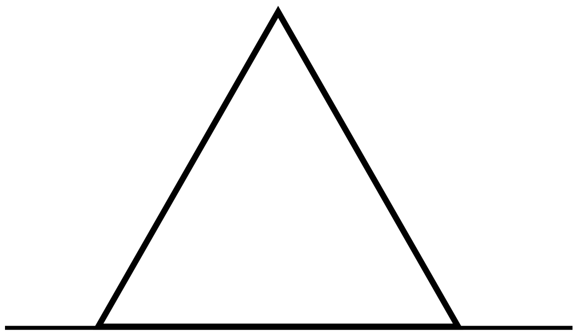
READ:	PROBLEMS TO DO (Show all work and support with explanations in your own words.):
Introduction	
Chapter 1	5, 7, 11, 14
Chapter 2 p. 11-16 (sec. 1 & 2 only)	4, 7, 11, 14, 15
Chapter 3	1, 4, 8, 13, 16, 20, 21a-e
Chapter 4	1a, 5, 6, 8&9 (graph w/calc.)
Chapter 5	2, 7(use sin(a-b) rule), 8, 9, 10
Chapter 6	2, 3c, d, 6, 8
Appendix: READ Polynomial, rational, exponential, logarithmic, trigonometric functions, inverse trig functions	DO: Attached Activities *Amusement Park *Reference Angles *Inverse Trig Functions *Trigonometry *Rational Functions *Rate of Change *Flash Cards

Amusement Park Rides

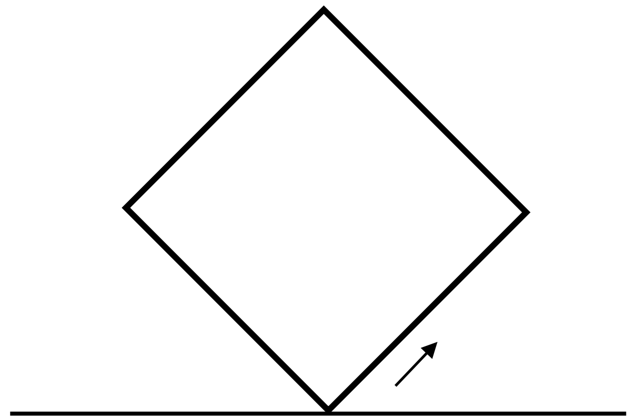
Name ____________________

Roller Coaster Rides

Imagine a roller coaster ride in the MathLand Amusement Park! There are two very strange roller coaster rides. Instead of making a loop, one roller coaster ride goes along a triangle track and the other on a square track, tilted at a 45° angle.



Cart →



Cart

Part 1

The triangle is an equilateral triangle, with each side equal to 60 feet. The square is also 60 feet on each side. The roller coaster car travels at 15 feet per second along the track. The car makes three complete loops before continuing along the track.

Your job is to create a **position versus time** and a **velocity versus time** graph for **<u>vertical height</u>** above the ground for each of these rides. Be sure to indicate in the diagram which direction the cart is traveling when it starts.

Amustment Park Rides

Name ____________________

Ferris Wheel Ride

Imagine sitting on a Ferris wheel as it is turning. We can think of this motion as circular since our body follows the circular path of the Ferris wheel. However, we can also think of this motion as a combination of vertical motion and horizontal motion. Some of the most useful application of the trigonometric functions lie in their ability to separate circular motion into its vertical and horizontal components.

Suppose a Ferris wheel with a 100 foot diameter makes one revolution every 24 seconds in a counterclockwise direction. The Ferris wheel is built so that the lowest seat on the wheel is 10 feet off the ground. This particular Ferris wheel has a boarding platform which is located at a height that is exactly level with the center (or hub) of the Ferris wheel. You take your seat level with the hub as the ride begins.

Draw a diagram of the Ferris wheel, with its lowest cart 10 feet off the ground and the boarding platform for a cart level with the center of the wheel.

Part 2

1. What is your height above the hub after 3 seconds (round you answer to the nearest thousandth)? How long does it take for you to reach the highest point? How high above the ground is that point?

2. Do you rise more in the first three seconds or in the next three seconds? Explain how you found your answer.

3. What is your height above the hub after 9 seconds? After 12 seconds? After 21 seconds? After 36 seconds?

4. Create a table that represents the relationship between time (t = 0, t = 3, t = 6, etc.) and height above the hub for <u>two revolutions</u> of the Ferris wheel. Explain how you determined these values.

5. Carefully sketch the graph of your function for two complete revolutions of the Ferris wheel **(on graph paper)** and write an equation that shows the height above the hub as a function of time. Check the equation with your table values.

6. If the ride lasts six minutes, what is the domain and range of your function in this context?

Part 3

7. How would your function change for each of the following situations? Be sure to show both **<u>an equation and a graph</u> (on graph paper)** for each situation. Describe how the original function was transformed.

   a. How would your function change if you wanted to know the height above the ground rather than above the hub?

   b. How would your function change if the Ferris wheel rotated twice times as fast? One-third as fast?

   c. How would your function change if the diameter of the Ferris wheel were 80 feet?

d. How would your function change if you board at the bottom of the Ferris wheel? (You still measure your height above the hub.) **Include a diagram of the Ferris wheel with your graph (on graph paper) and equation.**

e. How would your function change if the boarding platform is moved to the bottom of the Ferris wheel? Find your height above this new boarding platform as a function of time. **Include a diagram of the Ferris wheel with your graph (on graph paper) and equation**.

Part 4

8. Use your original equation and graph that shows the height above the hub as a function of time.

   a. Use the slope function on your calculator to find the <u>rate</u> at which the height is changing for two complete revolutions of the Ferris wheel. Carefully sketch the **graph** of the **rate** at which the height is changing as a function of time for two revolutions. Find an **equation** that approximates this graph.

   b. Is the velocity ever zero? If so, when does this occur? What is the height of the wheel when the velocity is zero?

   c. If the Ferris wheel rotated twice as fast, how would your position graph change? What would be the corresponding change in the velocity graph? Carefully sketch the new position **graph** and the **rate** of change graph. Find **equations** that approximate both the graphs.

Reference Angles, Trig and Inverse Trig Functions

a) y = sin (θ) if and only if θ = $\sin^{-1}$(y).
So, $f^{-1}(x) = \sin^{-1}(x) = \theta$ (the angle)

The domain of f(θ) = sin (θ) is all REAL numbers: $\theta \in \Re$.
The range of f(θ) is $-1 \le f(\theta) \le 1$.

The **domain** of the **inverse** function is $-1 \le x \le 1$.
The **range** of the **inverse** function is **limited.**

$$\frac{-\pi}{2} \le f^{-1}(x) \le \frac{\pi}{2} \qquad \textbf{OR} \qquad \frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$$

b) y = cos (θ) if and only if θ = $\cos^{-1}$(y).
So, $f^{-1}(x) = \cos^{-1}(x) = \theta$ (the angle)

The domain of f(θ) = cos (θ) is all REAL numbers: $\theta \in \Re$.
The range of f(θ) is $-1 \le f(\theta) \le 1$.

The **domain** of the **inverse** function is $-1 \le x \le 1$.
The **range** of the **inverse** function is **limited.**

$$0 \le f^{-1}(x) \le \pi \qquad \textbf{OR} \qquad 0 \le \theta \le \pi$$

c) y = tan (θ) if and only if θ = $\tan^{-1}$(y).
So, $f^{-1}(x) = \tan^{-1}(x) = \theta$ (the angle)

The domain of f(θ) = tan (θ)
is all REAL numbers excluding $\{\ldots -3\pi/2, -\pi/2, \pi/2, 3\pi/2, 5\pi/2, \ldots\}$ or
$\theta \neq (2k+1)\,\pi/2$ where k is an integer.
The range of f(θ) is all REAL numbers.

The **domain** of the **inverse** function is all REAL numbers.
The **range** of the **inverse** function is **limited.**

$$\frac{-\pi}{2} < f^{-1}(x) < \frac{\pi}{2} \qquad \textbf{OR} \qquad \frac{-\pi}{2} < \theta < \frac{\pi}{2}$$

Trig & Inverse Trig Functions Name ___________________

1. Graph the following on graph paper, from $[-2\pi, 2\pi]$. Use 6 blocks = π for the scale on the x-axis and 2 blocks = 1 for the y-axis. Be sure to accurately plot common reference angle points (x = $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \ldots$). Draw all asymptotes.

a) f(x) = sin (x)

b) g(x) = cos (x)

c) h(x) = tan (x)

d) k(x) = cot (x)

e) n(x) = sec (x)

f) p(x) = csc (x)

2. Use your calculator and sketch the **inverse functions** on the same axes as the original functions (from question #1). Pay attention to the **limited** range of each. Draw all asymptotes.

a) $y = \sin^{-1}(x)$

b) $y = \cos^{-1}(x)$

c) $y = \tan^{-1}(x)$

Inverse Trig Functions

Name ____________________

3. Use the **unit circle** to give the angle measure of each trigonometric expression. Give your answer in **<u>radian</u>** measure using π (no calculator). Remember the quadrants (range) for which each **<u>inverse</u>** function is defined.

a) $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ = ______

b) $\sin^{-1}(-1)$ = ______

c) $\cos^{-1}\left(\frac{1}{2}\right)$ = ______

d) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ = ______

e) $\cos^{-1}(0)$ = ______

f) $\sin^{-1}(1)$ = ______

m) $\cos^{-1}(-1)$ = ______

n) $\sin^{-1}\left(-\frac{1}{2}\right)$ = ______

o) $\tan^{-1}(-1)$ = ______

p) $\tan^{-1}(-\sqrt{3})$ = _____

q) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ = ______

g) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ = ______

h) $\cos^{-1}(1)$ = ______

i) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ = ______

j) $\cos^{-1}\left(-\frac{1}{2}\right)$ = ______

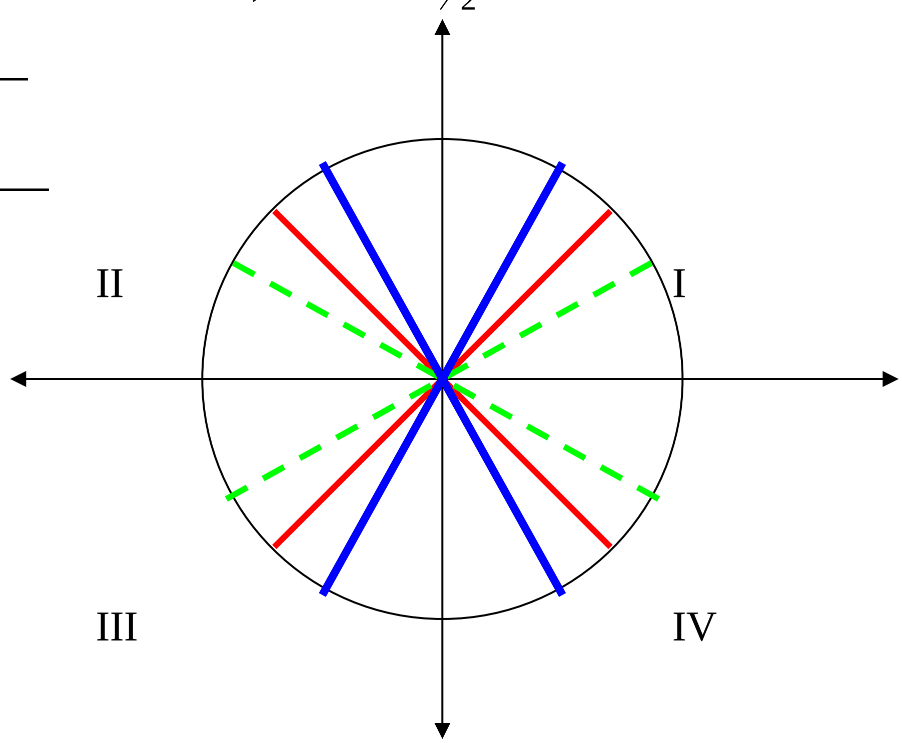
k) $\tan^{-1}(\sqrt{3})$ = ______

l) $\tan^{-1}(1)$ = ______

r) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ = ______

s) $\tan^{-1}(1)$ = ______

t) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ = ______



Inverse Trig Functions Name ____________________

4. A. If $\cos^{-1}\left(\frac{12}{13}\right) = \theta$ in Quadrant I, find: B. If $\tan^{-1}(\sqrt{5}) = \alpha$ in Quadrant I, find:

(No calculator - hint: Draw a right triangle for each.)

a) $\sin(\theta) =$ ____________________

b) $\cos(\theta) =$ ____________________

c) $\tan(\theta) =$ ____________________

d) $\cot(\theta) =$ ____________________

e) $\sec(\theta) =$ ____________________

f) $\csc(\theta) =$ ____________________

a) $\sin(\alpha) =$ ____________________

b) $\cos(\alpha) =$ ____________________

c) $\tan(\alpha) =$ ____________________

d) $\cot(\alpha) =$ ____________________

e) $\sec(\alpha) =$ ____________________

f) $\csc(\alpha) =$ ____________________

Trigonometry Name ____________________

Using the information (a)(b) = 0 if and only if a = 0 or b = 0, solve the following equation in the interval $[0, 2\pi)$. (No calculator.)

$2\sin(x)\cos(x) - \cos(x) = 0$

Any point on the unit circle has the coordinates $(\cos(\theta), \sin(\theta))$, so $x = \cos(\theta)$ and $y = \sin(\theta)$. From the Pythagorean theorem, $x^2 + y^2 = 1^2$ and substituting for x and y, then $\mathbf{\sin^2(\theta) + \cos^2(\theta) = 1}$.

Using the following identities, solve the following equations in the interval $[0, 2\pi)$. (No calculator.)

Pythagorean Identity: $\mathbf{\sin^2(x) + \cos^2(x) = 1} \rightarrow 1 - \sin^2(x) = \cos^2(x)$ **OR** $1 - \cos^2(x) = \sin^2(x)$

Solve for x, $[0, 2\pi)$: $\sin^2(x) - \cos^2(x) = 0$

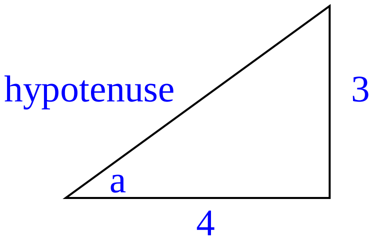
Solve for x, $[0, 2\pi)$: $2\cos(x)\tan(x) + \tan(x) - 2\cos(x) - 1 = 0$ (factor by grouping)

Trigonometry Name ____________________

Use the formulas given on the next page to calculate the value of the given expressions on the following pages **exactly** (no calculator). Follow the examples.

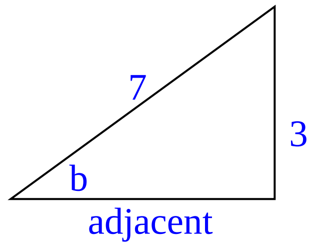
GIVEN: tan(a) = ¾ and csc(b) = 7/3, in Quadrant I, find cos(a – b), tan(2b) & cos(½a).

Using a right triangle (or the Pythagorean Identity), find the values of the other trig functions.



$hyp^2 = 3^2 + 4^2$

$hyp = 5$



$3^2 + adj^2 = 7^2$

$adj = 2\sqrt{10}$

From the first triangle, $\sin(a) = \frac{3}{5}$ and $\cos(a) = \frac{4}{5}$ and $\tan(a) = \frac{3}{4}$.

From the second triangle, $\sin(b) = \frac{3}{7}$ and $\cos(b) = \frac{2\sqrt{10}}{7}$ and $\tan(b) = \frac{3\sqrt{10}}{20}$.

FIND: cos (a – b), using the formula, cos (a – b) = cos(a)cos(b) + sin(a) sin(b).

Substitute into the formula and simplify.

$$\begin{aligned}\cos(a-b) &= \cos(a)\cos(b) + \sin(a)\sin(b)\\ &= (4/5)(2\sqrt{10}/7) + (3/5)(3/7)\\ &= (8\sqrt{10}+9)/35\end{aligned}$$

FIND: tan (2a), using the formula, $\tan(2a) = \dfrac{2\tan(a)}{1-\tan^2(a)}$.

$$\tan(2a) = \frac{2\tan(a)}{1-\tan^2(a)}$$

$$= \frac{2(3/4)}{1-(3/4)^2} = \frac{24}{7}$$

FIND: cos (½b), using the formula $\cos(\tfrac{1}{2}b) = \pm\sqrt{\dfrac{1+\cos(b)}{2}}$.

$$\cos(\tfrac{1}{2}b) = \sqrt{\frac{1+2\sqrt{10}/7}{2}} = \sqrt{\frac{7+2\sqrt{10}}{14}}$$ (Okay to leave in this form.)

Trigonometry Name ____________________

FORMULAS for sum & difference of angles, double angle and half-angle

sin(a + b) = sin(a)cos(b) + cos(a) sin(b)

sin(a – b) = sin(a)cos(b) – cos(a) sin(b)

cos(a + b) = cos(a)cos(b) – sin(a) sin(b)

cos(a – b) = cos(a)cos(b) + sin(a) sin(b)

tan(a + b) = $\frac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)}$

tan(a – b) = $\frac{\tan(a)-\tan(b)}{1+\tan(a)\tan(b)}$

sin(2a) = 2sin(a) cos(a)

cos(2a) = $\cos^2$(a) – $\sin^2$(a)

$\tan(2a)=\frac{2\tan(a)}{1-\tan^2(a)}$

sin(½a) = $\pm\sqrt{\frac{1-\cos(a)}{2}}$

cos(½a) = $\pm\sqrt{\frac{1+\cos(a)}{2}}$

tan(½a) = $\pm\sqrt{\frac{1-\cos(a)}{1+\cos(a)}}$

(The sign is determined by the quadrant.)

OR $\sin^2$(a) = $\frac{1-\cos(2a)}{2}$

$\cos^2$(a) = $\frac{1+\cos(2a)}{2}$ (Note: These are still double angle formulas.)

The ones in red are used most often in AP Calculus.

http://www.themathpage.com/atrig/trigonometric-identities.htm#double
Use this link to find the proof and explanation of the trigonometric identities above.

GIVEN: tan(a) = $\frac{2}{5}$ in Quadrant I and cos(b) = $\frac{4}{9}$ in Quadrant II
Set up the right triangles for $\angle a$ and $\angle b$. Find the lengths of the missing side.

Trigonometry Name ____________________

Using the triangles from the previous page find:

1. $\cos(a - b)$
2. $\sin(a + b)$
3. $\tan(a - b)$
4. $\sin(2b)$
5. $\cos(2a)$
6. $\cos(\frac{1}{2}b)$
7. $\sin^2(c)$, if $\cos(2c) = \frac{1}{3}$
8. $\cos^2(c)$, if $\cos(2c) = \frac{1}{3}$

Rational Functions Name ________________

GIVEN: $p(x) = x^2 - 1$ and $q(x) = x^3 - 1$

1. Graph $W(x) = \dfrac{p(x)}{q(x)}$ on graph paper. (Calculator window: Z4 to start)

2. Graph $Z(x) = \dfrac{q(x)}{p(x)}$ on graph paper. (Calculator window: Z4 to start)

3. Adjust your window as necessary. Check the table values. For each function, find the domain, range, end behavior model and end behavior asymptote, vertical asymptote(s), hole(s), local/absolute maximum(s) and minimum(s) and real root(s).

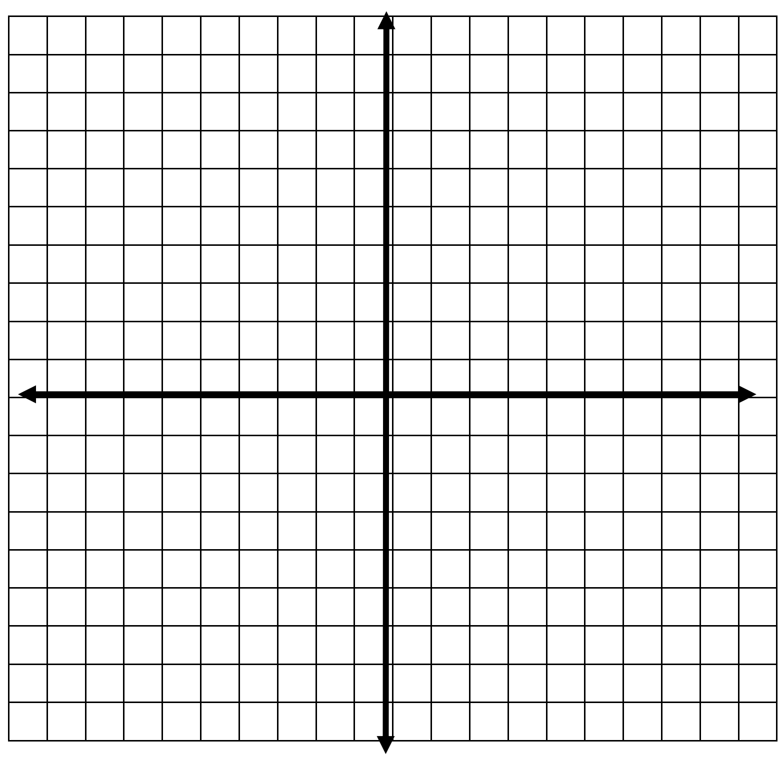
W(x)	Z(x)
Domain:	Domain:
Range:	Range:
VA:	VA:
Holes:	Holes:
EBM:	EBM:
EBA:	EBA:
Max:	Max:
Min:	Min:
Real Roots:	Real Roots:

Rate of Change Name ______________________

1. **GIVEN: $P(x) = 5x^3 - 2x^2 + 3x - 4$**
   Using the difference quotient and h = 0, find the equation of the
   A) velocity function, and
   B) acceleration function.

2. **Using the slope function and the table on your calculator, find the equation of the velocity function. Sketch the graphs of both functions below.**

   GIVEN: $P(x) = \ln(x)$



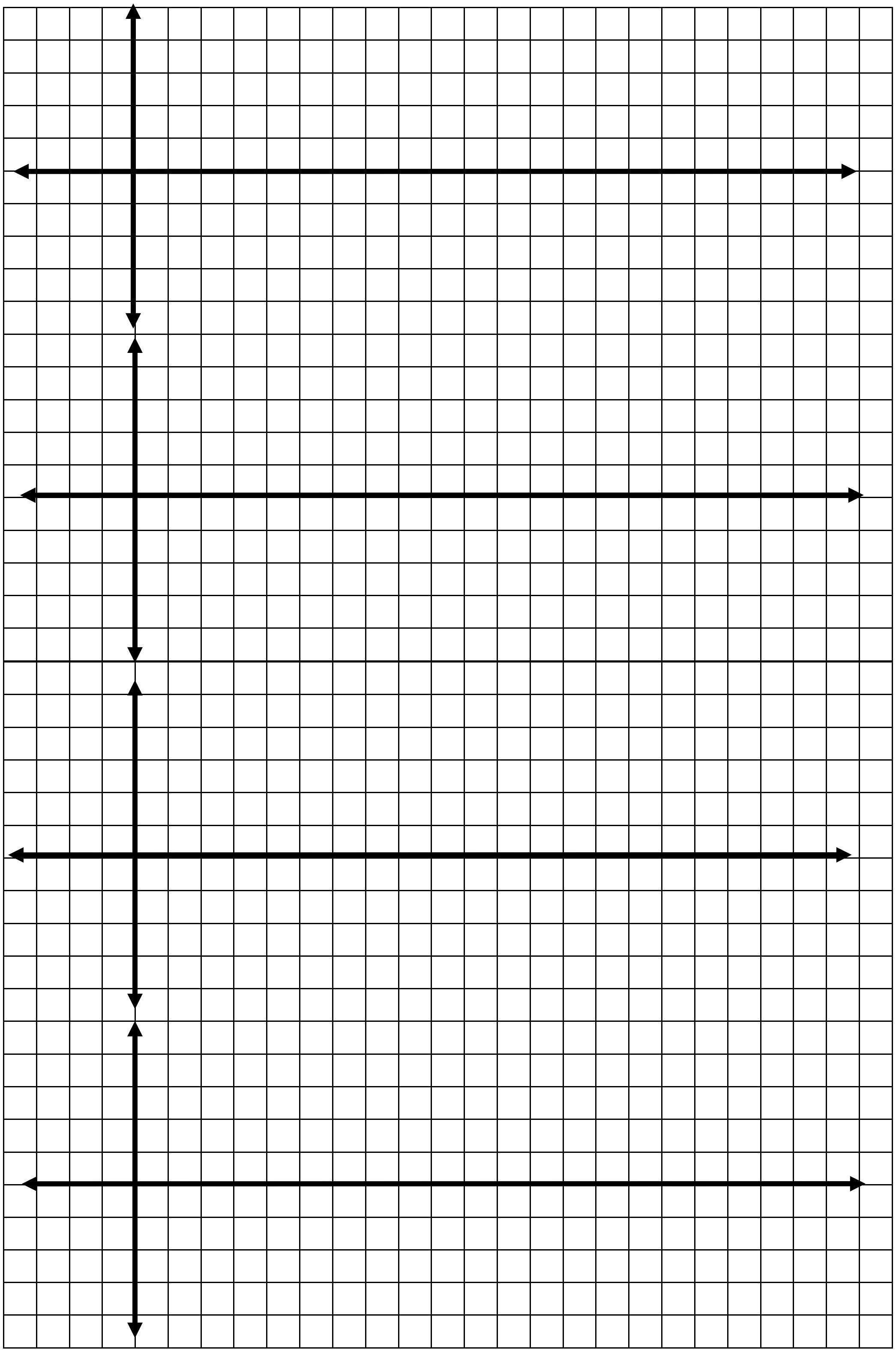
3. **GIVEN: P(x) = sin (x) Using the slope function on your calculator, find the equation of the A) velocity function,**

B) acceleration function, and

C) jerk function.

D) Sketch the graphs of <u>all four</u> functions below from $[0, 2\pi]$**.**

Use 6 blocks = π for the scale on the x-axis and 2 blocks = 1 for the y-axis.



P(x) = sin (x)

V(x) = ______________

Parent Functions and Transformations

Name ____________________

Create a deck of cards for matching. You will need to know these graphs.

A) Write the parent function on a card.
B) Draw its graph and state its domain and range on a second card.
C) Write the transformation described on a third card.
D) Draw the graph and write the equation for the transformation described on a fourth card.

You will need 60 cards total. Use a different color for each group.

Parent Functions	Transformations
1. $f(x) = x$	Vertical stretch of 3
2. $f(x) = \|x\|$	Horizontal shift left 3
3. $f(x) = [\![x]\!]$	Vertical shift down 1
4. $f(x) = x^2$	Vertical shrink of ½
5. $f(x) = x^3$	Horizontal shift right 1
6. $f(x) = \frac{1}{x}$	Horizontal shift left 1, vertical shift up 2
7. $f(x) = e^x$	Horizontal shift left 2
8. $f(x) = 2^x$	Vertical shrink of ½
9. $f(x) = ½^x$	Vertical stretch of 3 , vertical shift down 1
10. $f(x) = \log(x)$	Horizontal shift left 2
11. $f(x) = \ln(x)$	Horizontal shift right 3
12. $f(x) = \sin(x)$ from $[0, 2\pi]$	Horizontal shrink of 2
13. $f(x) = \cos(x)$ from $[0, 2\pi]$	Horizontal stretch of 2
14. $f(x) = \tan(x)$ from $[0, 2\pi]$	Reflection over the x axis
15. $x^2 + y^2 = 1$	Horizontal shift left 1, vertical shift up 1