Summer Work Packet for MPH Math Classes

Students going into
AP Calculus BC
Sept. 2018

Name:________________________

This packet is designed to help students stay current with their math skills.

Each math class expects a certain level of number sense, algebra sense and graph sense in order to be successful in the course.

These problems need to be completed on a separate sheet of paper (unless space has been provided) and turned in for a grade by September 7th. Be sure to show all work. If you have any questions, please email Mr. Ochs at jochs@mphschool.org or Mrs. Meehan at dmeehan@mphschool.org**.**

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The TI 84$^+$ calculator is good for use in AP Calculus. It does everything you are allowed to use it for on the AP Exam.

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AP CALCULUS BC

There are 4 main parts to this packet. Please follow the set of instructions for each part.

<u>Part I</u>

Instructions: Please watch the movie "Hidden Figures" and answer the following questions.

1. Characters: Give the first and last names of each person.

   Which of the three women is

   a. the engineer? ____

   b. the computer scientist? ____

   c. the mathematician? ____

   d. the head of the whole Space Task Group: ____________________________

   e. the engineer who is "second in command" in the Space Task Group?

   ____

2. Setting:

   a. Where does the movie take place? __________________________________

   b. Is the movie fiction, non-fiction or somewhere in between the two? ____________

   c. Who was the U.S. president at the time? ______________________________

   d. Was Martin Luther King Jr. still alive? ______________________________

   e. Give an example of how "colored" were treated differently than "whites."

   f. Give an example of how women were treated differently than men.

3. Katherine get an assignment to the Space Task Group.

   a. Why does she get the assignment? ____

   b. What kind of math can she do? ____

4. Mary wants to become an engineer but needs to take additional classes.

   a. Where does she take the classes? ____

   b. Who does she have to convince to let her enroll? ________________________________

5. Dorothy sneaks a book out of the library.

   a. What kind of book is it? ____

   b. Why does she have to sneak it out? ____

   c. Why does she need the book? ____

6.

   a. Who was the first Russian to orbit the earth? ____________________________________

   b. How many times did he orbit the earth? ____

   c. Who was the first American to orbit the earth? __________________________________

   d. How many times did he orbit the earth? ____

   e. Who orbited first, a Russian or an American? ____________________________________

7.

   a. What does IBM stand for? I _____________ B _______________ M _______________

   b. Who got the IBM? ____

   c. How many multiplications can the IBM compute per second? _____________________

8.
   a. Who figured out the landing coordinates? ____________________________________

   b. Using what method? ____

   c. Was this new math or old math? ____

9. Who knocked down the "colored ladies room" sign? __________________________
   Why?

10.
   a. What parting gift does Katherine get right before her wedding? ___________________

   b. What is the significance of the gift? ____

11.
   a. What happens right before John Glenn's lift off? ________________________________

   b. Who does John Glenn ask to check the coordinates? ____________________________

12.
   a. Who becomes the supervisor at the end of the movie? ___________________________

   b. What does Mrs. Mitchell call her at that time and what is the significance of this?

13.
   a. How many years were Katherine and Jim married? ______________________________

   b. What happened to Katherine's first husband? __________________________________

14. When Mary goes to night school, where is the “colored section” of the classroom?

____

15. State the number of children each woman has.

   a. Katherine ____

   b. Mary ____

   c. Dorothy ____

16.

   a. How many times is John Glenn scheduled to orbit the earth? ______________________

   b. How long is it supposed to take? ____

   c. How many times does he actually orbit the earth? ________________________________

   d. Why did they have to bring him down early? ____________________________________

17. From where did John Glenn take off? ____

18. Where did John Glen land? ____

19.

   a. What does Paul bring Katherine at the end of the movie? _________________________

   b. What is the significance of this? ____

Part II

Instructions:

1. Go to https://www.khanacademy.org/
2. Click on "Learners, start here."
3. Under **Math by subject,** click "AP Calculus BC."
4. Scroll down until you reach the section that says "Sequences & series intro."

You will complete lessons and practice exercises from topics *off the menu.* All the topics with the videos (lessons) you are responsible for watching are listed below, followed by their corresponding practice exercises.

Complete the practice exercise on a separate sheet of paper. These will be collected the first week of school. Please let me know if you have any questions.

Sequences Review:

1. Videos: "Sequences Intro", "Worked Example Sequence: Explicit Formula", "Worked Example Sequence: Implicit Formula"
2. Practice: "Sequences Review"

Infinite Sequences:

1. Videos: "Convergent and Divergent Sequences", "Worked Example: Sequences Convergent/Divergent"
2. Practice: "Sequence Convergent/Divergent"

Series Review:

1. Videos: "Sigma Notation for Sums", "Worked Example: Sigma Notation", "Worked Example: Sigma Notation ($n\geq2$)"
2. Practice: "Sigma Notation Intro"

Partial Sums:

1. Videos: "Partial Sums Intro", Partial Sums; Formula for n^{th} term from partial sum", "Partial Sums: term value form partial sum"
2. Practice: "Partial Sums Intro",

Part III

Please make sure to read this carefully and complete the examples. This will be collected the first week of school.

Consider the function y = sin(x). We know that for all x in R, $-1 \le \sin(x) \le 1$. Recall, this is equivalent to saying that $|\sin(x)| \le 1$ for all x in R.

In this case we say that 1 is a **bound for** sin(x) **on** R. Intuitively, a bound for a function is a real number, M such that the absolute value of the function never exceeds M.

Definition: Let f be a function defined on a closed interval [a, b] and let M be a positive real number. We say M is a **bound** of $f(x)$ if

$$|f(x)| \le M \text{ for all } x \text{ in } [a, b] \text{ and } M \text{ is } \ge 0.$$

An important point to notice is that bounds are not unique. While it is true that $|\sin(x)| \le 1$ for all x in R, it is also true that $|\sin(x)| \le 2$, $|\sin(x)| \le 3$, … , $|\sin(x)| \le 100$, etc. However, 1 is the smallest bound so we think of it as the "best" in some sense since anything smaller than 1 is not a bound for sin(x). Let's work out some explicit examples to see what this means.

Example 1: Let $f(x) = \sin(x)$. State three bounds for $f(x)$ and determine the smallest bound for $f(x)$.

Solution. Since $|f(x)| \le 1$ for all x in R, then 3 bounds for $f(x)$ could be 1, 5 and 17. But as stated above, the smallest bound for $f(x)$ is 1. This can be seen by looking at the graph of sin(x).

Example 2: Let $f(x) = 2\sin(x) + 1$. State three bounds for $f(x)$ and determine the best (smallest) bound for $f(x)$.

Solution: Since $|f(x)| \le 3$ for all x in R, then 3 bounds for $f(x)$ could be 3, 5 and 10. But the smallest bound for $f(x)$ is 3. This can be seen by looking at its graph.

Now consider the function $f(x) = x$. This linear function does not have a bound on the whole real number line. In other words, there is no number M such that $|x| \le M$ for all x in R. Graphically we understand this because $|x|$ goes to infinity for both positive and negative x.

But when we limit the domain values to a closed interval, for example [1, 3], the y-values are limited as well. So in general we care mostly about continuous functions defined on a closed interval because these functions can always be bounded. In practice, the Extreme Value Theorem (EVT) gives us a procedure to find a bound.

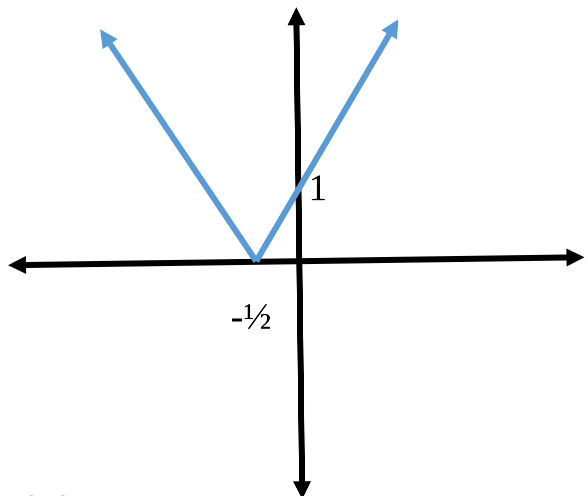
Theorem 1 EVT: Let *f(x)* be a continuous function defined on a closed interval [a, b]. Then, *f(x)* attains both a maximum and a minimum on [a, b].

Our goal is to find bounds for the absolute value of functions. We will do this by using the Extreme Value Theorem and the following theorem.

Theorem 2: If *f(x)* is a continuous function on the closed interval [a, b], then | *f(x)*| is also continuous on [a, b].

Combining these two, if *f(x)* is continuous on [a, b], then | *f(x)*| attains a maximum on [a, b].

Example 3: In this example we will be finding a bound for *f(x)* = 2x + 1 on the closed interval [–2 , 2]. The first step is to graph |*f(x)*| = |2x + 1|.



Next find the maximum of |*f(x)*| on the given interval. In this example, since 5 is the largest y-value attained on [–2 , 2], we have that

$$|f(x)| \leq \text{ for all } x \text{ in } [-2\ ,\ 2].$$

So, *M* = 5 and we have found our bound for *f(x)* on [–2 , 2].

Please complete the following questions on a separate sheet of paper. This will be collected the first week of school.

Find a bound for each of the following functions on the given intervals. Provide a graph for *f(x)* and |*f(x)*|.

1. $f(x) = e^x$ on [½, 1]

2. *f(x)* = 1/x on [–3 , –2]

3. *f(x)* = ln(x) on [¼, 1] and [1, 2]

Part IV

Barron's AP Calculus Flash Cards, 2nd Edition by David Bock M.S will need to be purchased to complete the summer assignment for AP Calculus BC **ISBN-13:** 978-1438074009 or **ISBN-10:** 143807400X. To be successful in AP Calculus BC you must be very confident and comfortable with concepts and topics learned in AP Calculus AB. We will spend a minimal amount of time going over material previous learned in AP Calculus AB.

This set of Calculus flash cards will help you review the key terms, facts and equations that you need to know to be successful in this upcoming year. Please read through all of the cards marked AB. You will need to know/memorize ALL of the cards marked AB. You will find that many of the flashcards are bi-directional – you can study by looking at either side of the card and thinking about what must be on the reverse side. Make sure you feel comfortable recalling information in both directions.

In the set of cards you will also find 100 sample problems with answers on the reverse side. Over the course of the summer you will need to work through **ONLY** the sample problems that are marked **AB**. These problems need to be completed on a separate sheet of paper by the first day of class. Be sure to show all work.

Students can expect a quiz/test on the material contained in the flashcards and covered in the AP Calculus AB curriculum during the first marking period.

Students will need a TI-84+ calculator for AP Calculus BC.

Good luck, have fun and please contact me or Mrs. Meehan with any concerns or problems!

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