Summer Work Packet for MPH Math Classes

Students going into Geometry AC Sept. 2019

Name: ___________________________

This packet is designed to help students stay current with their math skills.

Each math class expects a certain level of number sense, algebra sense and graph sense in order to be successful in the course.

Complete these problems in the space provided. It will be handed in for a grade on the first week of school. Be sure to show all work.

If you have any questions, please email Mrs. Meehan at dmeehan@mphschool.org . The beginning work should be review. The questions on LOGIC require you to read the notes to learn about the notation and vocabulary and then answer the questions. The NUMBER DEVIL will be used throughout the year.

You will need to read the NUMBER DEVIL, by Hans Magnus Enzenberger and have a TI-84+ calculator, a notebook, three 2 pocket folders with tabs, a compass and a protractor for this class.

THE NUMBER DEVIL by Hans Magnus Enzenberger

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Math is about more than numbers. It's also about patterns and making connections. This year you will be developing, analyzing and writing about mathematics using chapters from THE NUMBER DEVIL as the starting point. To be able to complete your essays next year, you will need to read the entire book this summer. As you are reading keep in mind the patterns discussed in the book and the non-mathematical phraseology (vroom, rutabagas, etc). ENJOY!

1. **The Properties of Equality:**

   If a = b, then ... a + c = b + c   a – c = b – c $a \times c = b \times c$ a/c = b/c.

   These properties are used algebraically for solving equations. In Geometry, they will be used with segments, angles and arcs.

2. **Substitution Property:** If a = c and b = c, then a = b.

   This property is used algebraically to evaluate expressions and **check** equations. In Geometry, it will be used to prove segments, angles and arcs equal.

3. **Distributive Property:** a(b + c) = ab + ac and ab + ac = a(b + c) (factoring out "a")

   This property is used algebraically to simplify expressions and combine like terms, or for example, to factor a quadratic in order to solve for x. In Geometry, it will also be used to solve equations.

4. **Radicals** (square roots) and quadratic expressions are likely to show up when solving a problem. In Geometry, these are common in problems that deal with Similar and Right Triangles, especially with the Geometric Mean and the Pythagorean Theorem.

   $a\sqrt{b} \cdot c\sqrt{d} = ac\sqrt{bd}$ $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{a/b}$ $\sqrt{a} \cdot \sqrt{a} = \sqrt{a^2} = |a|$

   $a\sqrt{b} + c\sqrt{b} = (a+c)\sqrt{b}$

   Remember, proper form for radical expressions means:
   a. No perfect square factor under the radical. For example, $\sqrt{45} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$
   b. No fractions or decimals may be left under the radical. For example, $\sqrt{3/4} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$
   c. No radical may be left in the denominator of a fraction. For example,
   $\sqrt{3/7} = \frac{\sqrt{3}}{\sqrt{7}} = \frac{\sqrt{3}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{21}}{7}$. Another example, $\frac{15\sqrt{75}}{20\sqrt{21}} = \frac{3\sqrt{25}\sqrt{3}}{4\sqrt{7}\sqrt{3}} = \frac{3 \cdot 5}{4\sqrt{7}} = \frac{15\sqrt{7}}{28}$.

5. It is important to be able to **translate words into mathematics**. This allows you to take information describing the relationship between shapes and turn it into mathematical symbols to solve the problem. These situations can come in many different styles. In algebra, they tend to be referred to as "word problems." In Geometry, the information relates to the various shapes and their specific characteristics.

6. Though we won't specifically use **Logic** notation and all its laws, when we study Geometry we will be proving many theorems. To do this we will set up a series of steps, each of which will be supported by a theorem, definition or postulate. This is similar to creating logic arguments and determining if the stated conclusion is valid and supported by a Law of Reasoning. These Laws of Reasoning are listed with the problems on logic. You will need to follow the examples and check out the symbols.

YOU MAY DO ALL WORK ON THESE PAGES.

Solve each equation. (This is using the Properties of Equality.) Check your answer. (This is using the Substitution Property.)

1. $\frac{1}{2}x - 5 = \frac{3}{4}x + 7$

2. $5(2x - 3) = \frac{2}{3}(12x - 15)$

3. $\frac{7x-4}{6} = \frac{10x+3}{5}$

4. $3(x + 2) - 4(2x - 5) = 10(x - 3)$

Simplify each expression.

5. $\dfrac{4(-3x^3y^5)}{2(xy^2)^3}$

6. $(3y - 4x + 2z) - (z - y + 8x) - (5x + 6z - 9y)$

7. $2x - 3(x - 4) + 5(6x - 1)$

8. $\dfrac{36x^2y}{45xy^4}$

9. $(a^3b^4c^7)(3a^3b^6c^4)^2$

Use the Distributive Property (FOIL) to write as an expression without parentheses in <u>simplest</u> form.

10. $(x - 9)(x - 8)$

11. $(x + 7)(x - 7)$

12. $(6x - 5)(x - 9)$

13. $(x + 3)(3x - 8)$

14. $(5x - 4)(5x + 4)$

15. $(3x + 5)(3x + 5)$

16. $(2x - 11)(2x - 11)$

17. $(2x + 5)(3x - 10)$

Factor completely, then solve for x algebraically.

18. $x^2 - 10x - 24 = 0$

19. $2x^2 + 7x - 4 = 0$

20. $3x^2 - 10x + 8 = 0$

21. $3x^2 + 10x + 7 = 0$

22. $x^2 - 16x + 60 = 0$

23. $3x^2 - 16x + 20 = 0$

Solve each of the problems below. Be sure to write the area formula for each.

24. Find the perimeter and area of a square whose side measures $4\sqrt{6}$.

25. Find the area and circumference of a circle with a diameter of 60. Leave π in your answer.

26. The area of a circle is 169π cm^2. Find the length of the diameter.

Simplify the radical expressions. Leave each in its best radical form (no decimal equivalents). Use the notes at the beginning of the packet to help you.

27. $5\sqrt{12}\cdot 3\sqrt{6}$

32. $\dfrac{\sqrt{180}}{2\sqrt{18}}$

28. $\dfrac{6\sqrt{1000}}{5}$

33. $\dfrac{\sqrt{40}}{\sqrt{220}}$

29. $\dfrac{\sqrt{12}}{\sqrt{75}}$

34. $\dfrac{6}{\sqrt{48}}$

30. $\dfrac{-22}{\sqrt{121}}$

34. $\sqrt{\dfrac{3}{12}}$

31. $18-2\sqrt{18}$

35. $\dfrac{14\sqrt{72}}{3\sqrt{28}}$

Write the equation of a line in in slope-intercept form: y = mx + b, where m is the slope and b is the y-intercept.

37 . m = 5, (0, –6)

38. m = –½, (6, –8)

39. (–6, –8) and (2, 6)

40. Parallel to the line y = 5x – 7 that goes through the point (–2, –4)

41. Perpendicular to the line y – 2x = 12 and goes through the point (–6 , 2)

LAWS OF LOGIC

$\wedge$ means AND **$\vee$ means OR**

$\rightarrow$ means implies or If..., then... **~ means NOT (opposite)**

p and q are used to represent a simple statement. p: It is sunny outside. q: It is warm outside.

$p \wedge q$ represents "It is sunny outside and it is warm outside."
$p \vee q$ represents "It is sunny outside or it is warm outside."
$p \rightarrow q$ represents "If it is sunny outside, then it is warm outside."
Or, "It is sunny outside implies it is warm outside."
~p represents "It is not sunny outside." (If p is true, ~p is false. If p is false, ~p is true.)
~(~p) = p

p can be true or false. q can be true of false. There are 4 possible combinations of p and q.

p	**q**	**$p \wedge q$**
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**F**

"p and q" is only TRUE when BOTH p and q are true.

p	**q**	**$p \vee q$**
T	**T**	**T**
T	**F**	**T**
F	**T**	**T**
F	**F**	**F**

"p or q" is only FALSE when BOTH p and q are false.

p	**q**	**$p \rightarrow q$**
T	**T**	**T**
T	**F**	**F**
F	**T**	**T**
F	**F**	**T**

"p implies q" is only FALSE when a true statement implies a false one.
(p is the hypothesis and q is the conclusion.)

Conditional statements, $p \rightarrow q$, have related conditionals statements.
converse: $q \rightarrow p$
inverse: $\sim p \rightarrow \sim q$
contrapositive: $\sim q \rightarrow \sim p$

Use the information above to do the following.

p: PQRS is a square. q: PQRS is a rectangle.

42. Write $p \wedge q$ in words.

43. Write $p \vee q$ in words.

44. Write $p \rightarrow q$ in words.

45. Write $\sim p$ in words.

46. Explain why "p: PQRS is a square" is not the <u>opposite</u> of "q: PQRS is a rectangle."

47. Write the converse, $q \rightarrow p$, in words.

48. Write the inverse, $\sim p \rightarrow \sim q$, in words.

49. Write the contrapositive, $\sim q \rightarrow \sim p$, in words.

50. Which of these 4 conditional statements (#44, 47, 48, 49) are true? Explain.

Laws of Logic (Look for the pattern of the argument.)

A ***premise*,** represented as P_1, or P_2 etc., is <u>given</u> as true. The ***conclusion*** is represented as **C**.

Law of Detachment: If $(p \rightarrow q)$ is true and p is true, then q is true.
P_1: $p \rightarrow q$
P_2: p
Therefore, C: q

Law of Contrapositive Inference: If $(p \rightarrow q)$ is true and ~q is true, then ~p is true.
P_1: $p \rightarrow q$
P_2: ~q
Therefore, C: ~p

Law of Disjunctive Inference: If $(p \vee q)$ is true and ~p is true, then q is true.

P_1: $p \vee q$	OR	P_1: $p \vee q$
P_2: ~p		P_2: ~q
Therefore, C: q		Therefore, C: p

Law of Syllogism: If $(p \rightarrow q)$ is true and $(q \rightarrow r)$ is true, then $(p \rightarrow r)$ is true.
P_1: $p \rightarrow q$
P_2: $q \rightarrow r$
Therefore, C: $p \rightarrow r$

Law of Contrapositive Equivalence: $p \rightarrow q = \sim q \rightarrow \sim p$
Example: If $x + 4 = 7$, then $x = 3$. Contrapositive: If $x \neq 3$, then $x + 4 \neq 7$. Both are TRUE.

Examples of how an argument looks and the law that supports it:

P_1: If I study for the test, then I will get an A.
P_2: I studied for the test.

Therefore, I will get an A **Law of Detachment**

P_1: If I study for the test, then I will get an A.
P_2: I didn't get an A.

Therefore, I didn't study for the test. **Law of Contrapositive Inference**

P_1: I'll have pizza for lunch or I'll have a salad for lunch.
P_2: I'm not having pizza for lunch.

Therefore, I'll have a salad for lunch. **Law of Disjunctive Inference**

P_1: If I clean my room, then I can go to the movies.
P_2: If I go to the movies, then my friends will meet me at the mall.

Therefore, if I clean my room, then my friends will meet me at the mall. **Law of Syllogism**

P_1: Conditional statement: If I stay up late, then I will be tired in the morning.

Therefore, Contrapositive statement: If I am <u>not</u> tired in the morning, then I did <u>not</u> stay up late. **Law of Contrapositive Equivalence**

State a law of logic listed above that can be used to draw a valid conclusion. Follow the pattern of the rules.

EXAMPLE: P_1: ~k → ~h
P_2: h

Conclusion: k

Law: Contrapostive Inference (opposite of the conclusion (~h) implies the opposite of the hypothesis (~k).)

51. P_1: f → g
P_2: f

Conclusion: ____________________ Law: ____________________

52. P_1: ~d → ~w
P_2: ~d

Conclusion: ____________________ Law: ____________________

53. P_1: r → ~g
P_2: g

Conclusion: ____________________ Law: ____________________

54. P_1: h ∨ d
P_2: ~h

Conclusion: ____________________ Law: ____________________

55. P_1: c → k
P_2: k → ~m

Conclusion: ____________________ Law: ____________________

Show that the indicated conclusion follows from the given premises. Use the rules of logic listed above to create a valid argument.

56. P_1: $p \to r$
P_2: $\sim p \to d$
Therefore, C: $\sim r \to d$

Steps	Reasons
1. $p \to r$	1. Premise 1
2. $\sim r \to \sim p$	2. ______________________________ Which rule lets you go from 1 to 2?
3. $\sim p \to d$	3. Premise 2
4. $\sim r \to d$	4. ______________________________ Which rule combines steps 2 and 3?

57. P_1: $\sim r \to p$
P_2: $\sim s$
P_3: $r \to s$
Therefore, C: p

Steps	Reasons
1. $r \to s$	1. Premise 1
2. $\sim s$	2. Premise 3
3. $\sim r$	3. ______________________________ Which rule combines steps 1 and 2?
4. $\sim r \to p$	4. Premise 2
5. p	5. ______________________________ Which rule combines steps 3 and 4?

Now you try one. Put the statements in order to make a valid conclusion. Use the laws of logic. If you are using one of the given statements, write "premise #__" as your reason for why it is true. There is more than one way to do it.

58. P_1: $p \rightarrow z$
P_2: $z \rightarrow \sim k$
P_3: p
Therefore, C: $\sim k$

Steps	Reasons
1. ____________	1. ____________________________
2. ____________	2. ____________________________
3. ____________	3. ____________________________
4. ____________	4. ____________________________
5. ____________	5. ____________________________

Don't forget….

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