Summer Work Packet for MPH Math Classes

Students going into Geometry C Sept. 2021

Name: ___________________________

These problems need to be completed, in the space provided by September 17th. Be sure to show all your work. We will check this assignment in class. Remember, it's about the process, not just the answer.

Please try to pace yourself throughout the summer. Completing one problem every day, or a page of problems every week, is a nice way to work through the packet.

If you have any questions, please email Mrs. Meehan at dmeehan@mphschool.org. The beginning work should be review. The questions on LOGIC require you to read the notes to learn about the notation and vocabulary, and then answer the questions.

Math is about more than numbers. It is also about patterns and making connections. This year you will be developing, analyzing, and writing about mathematics using chapters from THE NUMBER DEVIL as the starting point. You may read the book this summer, or as we go along during the school year. We will be doing various activities related to the book each quarter.

THE NUMBER DEVIL by Hans Magnus Enzenberger
Publisher: Holt Paperbacks (May 1, 2000)
ISBN-10: 0805062998 **ISBN-13:** 978-0805062991

Supplies:
TI-84+ calculator (Please bring in the points off the packaging if you buy a new one.)
Notebook/binder (To take notes and save handouts)
Three 2 pocket folders with tabs
Compass (Ones with a wheel, or screw to tighten tend to work better and last longer.)
Protractor
Colored pencils

1. **The Properties of Equality:**

If a = b, then ... a + c = b + c a – c = b – c $a \times c = b \times c$ a/c = b/c.

These properties are used algebraically for solving equations. In Geometry, they will be used with segments, angles and arcs.

2. **Substitution Property:** If a = c and b = c, then a = b.

This property is used algebraically to evaluate expressions and **check** equations. In Geometry, it will be used to prove segments, angles and arcs equal.

3. **Distributive Property:** a(b + c) = ab + ac and ab + ac = a(b + c) (factoring out "a")

This property is used algebraically to simplify expressions and combine like terms, or for example, to factor a quadratic in order to solve for x. In Geometry, it will also be used to solve equations.

4. **Radicals** (square roots) and quadratic expressions are likely to show up when solving a problem. In Geometry, these are common in problems that deal with Similar and Right Triangles, especially with the Geometric Mean and the Pythagorean Theorem.

$$a\sqrt{b}\cdot c\sqrt{d} = ac\sqrt{bd} \qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{a/b} \qquad \sqrt{a}\cdot\sqrt{a} = \sqrt{a^2} = |\,a\,|$$

$$a\sqrt{b} + c\sqrt{b} = (a+c)\sqrt{b}$$

Remember, proper form for radical expressions means:

a. No perfect square factor under the radical. For example, $\sqrt{45} = \sqrt{9}\cdot\sqrt{5} = 3\sqrt{5}$

b. No fractions or decimals may be left under the radical. For example, $\sqrt{3/4} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$

c. No radical may be left in the denominator of a fraction. For example,

$\sqrt{3/7} = \frac{\sqrt{3}}{\sqrt{7}} = \frac{\sqrt{3}}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{21}}{7}$. Another example, $\frac{15\sqrt{75}}{20\sqrt{21}} = \frac{3\sqrt{25}\sqrt{3}}{4\sqrt{7}\sqrt{3}} = \frac{3\cdot 5}{4\sqrt{7}} = \frac{15\sqrt{7}}{28}$.

5. It is important to be able to **translate words into mathematics**. This allows you to take information describing the relationship between shapes and turn it into mathematical symbols to solve the problem. These situations can come in many different styles. In algebra, they tend to be referred to as "word problems." In Geometry, the information relates to the various shapes and their specific characteristics.

6. Though we won't specifically use **Logic** notation and all its laws, when we study Geometry we will be proving many theorems. To do this we will set up a series of steps, each of which will be supported by a theorem, definition or postulate. This is similar to creating logic arguments and determining if the stated conclusion is valid and supported by a Law of Reasoning. These Laws of Reasoning are listed with the problems on logic. You will need to follow the examples and check out the symbols.

YOU MAY DO ALL WORK ON THESE PAGES.

Solve each equation. (This is using the Properties of Equality.)
<u>Check</u> your answers. (This is using the Substitution Property.)

1. $\frac{1}{2}x + 5 = \frac{3}{4}x - 7$

3. $5(3x - 2) = \frac{3}{4}(12x - 16)$

2. $4(2x - 5) - 3(x + 2) = -10(x - 3)$

4. $\frac{7x-4}{6} = \frac{10x+3}{5}$

Simplify each expression.

5. $\frac{24(-3x^8y^5)}{-12(xy^2)^3}$

6. $(2y - 3x + 4z) - (z - y - 8x) - (3x + 4z - 9y)$

7. $2x + 3(x - 4) - 5(6x - 7)$

8. $\frac{36x^2y}{45xy^4} \cdot (-5x^3y^2)^0$

9. $(a^{-3}b^4c^{-5})(-6a^3b^{-6}c^4)^2$

Use the Distributive Property (FOIL) to write as an expression without parentheses in <u>simplest</u> form.

10. $(x - 6)(x - 7)$

11. $(5x - 6)(x - 9)$

12. $(x + 4)(3x - 8)$

13. $(3x - 5)(3x + 5)$

14. $(4x + 9)(4x + 9)$

15. $(5x + 2)(3x - 10)$

<u>Factor</u> completely, then solve for x algebraically.

16. $x^2 - 11x + 24 = 0$

17. $2x^2 - 7x - 4 = 0$

18. $3x^2 - 10x - 8 = 0$

19. $3x^2 + 10x + 7 = 0$

Solve each of the problems below. Be sure to write the area formula for each.

20. Find the perimeter and area of a square whose side measures $6\sqrt{3}$.

21. Find the area and circumference of a circle with a diameter of 20. Leave π in your answer.

22. The area of a circle is 225π cm^2. Find the length of the diameter.

Simplify the radical expressions. Leave each in its best radical form (no decimal equivalents). Use the notes at the beginning of the packet to help you.

23. $(18 - 2\sqrt{18}) + (12 + 3\sqrt{50}\ )$

26. $\sqrt{\frac{5}{20}}$

24. $\frac{\sqrt{75}}{\sqrt{12}}$

27. $5\sqrt{6} \cdot 3\sqrt{24}$

25. $\frac{14\sqrt{72}}{3\sqrt{28}}$

Write the equation of a line in in slope-intercept form: y = mx + b, where m is the slope and b is the y-intercept.

28. $m = 6$, $(0, –5)$

29. $m = –\frac{1}{2}$, $(8, –6)$

30. $(–6, –8)$ and $(1, 6)$

31. Parallel to the line $y = 7x – 5$ that goes through the point $(–2, –5)$

32. Perpendicular to the line $3y – 2x = 12$ and goes through the point $(–6, 2)$

LAWS OF LOGIC

$\wedge$ means AND | **$\vee$ means OR**

$\rightarrow$ means implies or If..., then... | **~ means NOT (opposite)**

p and q are used to represent a simple statement. p: It is sunny outside. q: It is warm outside.

p $\wedge$ q represents "It is sunny outside **and** it is warm outside."
p $\vee$ q represents "It is sunny outside **or** it is warm outside."
p $\rightarrow$ q represents "**If** it is sunny outside, **then** it is warm outside."
Or, "It is sunny outside **implies** it is warm outside."
~p represents "It is **not** sunny outside." (If p is true, ~p is false. If p is false, ~p is true.)
~(~p) = p

p can be true or false. q can be true of false. There are 4 possible combinations of p and q.

p	**q**	**p $\wedge$ q**
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**F**

"p and q" is TRUE only when BOTH p and q are true.

p	**q**	**p $\vee$ q**
T	**T**	**T**
T	**F**	**T**
F	**T**	**T**
F	**F**	**F**

"p or q" is FALSE only when BOTH p and q are false.

p	**q**	**p $\rightarrow$ q**
T	**T**	**T**
T	**F**	**F**
F	**T**	**T**
F	**F**	**T**

"p implies q" is FALSE only when a true statement implies a false one.
(p is the hypothesis and q is the conclusion.)

A conditional statement (p $\rightarrow$ q) has related conditionals statements:
converse: q $\rightarrow$ p (switch the hypothesis and conclusion)
inverse: ~p $\rightarrow$ ~q (negate the hypothesis and conclusion)
contrapositive: ~q $\rightarrow$ ~p (switch and negate the hypothesis and conclusion)

Use the information above to do the following.

p: JKLM is a rectangle. q: JKLM is a square.

33. Write $p \wedge q$ in words.

34. Write $p \vee q$ in words.

35. Write $p \rightarrow q$ in words.

36. Write $\sim p$ in words.

37. Explain why "p: JKLM is a rectangle" is **not** the opposite of "q: JKLM is a square.

38. Write the converse ($q \rightarrow p$) in words.

39. Write the inverse ($\sim p \rightarrow \sim q$) in words.

40. Write the contrapositive ($\sim q \rightarrow \sim p$) in words.

41. Which of these 4 conditional statements (#35, 38, 39, 40) are true? Explain.

Laws of Logic (Look for the pattern of the argument.)

A ***premise***, represented as P_1, or P_2 etc., is given as true. The ***conclusion*** is represented as **C**.

Law of Detachment: If ($p \rightarrow q$) is true and p is true, then q is true.
P_1: $p \rightarrow q$
P_2: p
Therefore, C: q

Law of Contrapositive Inference: If ($p \rightarrow q$) is true and ~q is true, then ~p is true.
P_1: $p \rightarrow q$
P_2: ~q
Therefore, C: ~p

Law of Disjunctive Inference: If ($p \vee q$) is true and ~p is true, then q is true.

P_1: $p \vee q$	OR	P_1: $p \vee q$
P_2: ~p		P_2: ~q
Therefore, C: q		Therefore, C: p

Law of Syllogism: If ($p \rightarrow q$) is true and ($q \rightarrow r$) is true, then ($p \rightarrow r$) is true.
P_1: $p \rightarrow q$
P_2: $q \rightarrow r$
Therefore, C: $p \rightarrow r$

Law of Contrapositive Equivalence: $p \rightarrow q = {\sim}q \rightarrow {\sim}p$
Example: Conditional: If $x + 4 = 7$, then $x = 3$.
Contrapositive: If $x \neq 3$, then $x + 4 \neq 7$.
Both are TRUE.

Examples of how an argument looks and the law that supports it:

P_1: If I study for the test, then I will get an A. **$t \rightarrow a$**
P_2: I studied for the test. **t**

Therefore, I will get an A. **a** **Law of Detachment**

P_1: If I study for the test, then I will get an A. **$t \rightarrow a$**
P_2: I didn't get an A. **$\sim a$**

Therefore, I didn't study for the test. **$\sim t$** **Law of Contrapositive Inference**

P_1: I'll have pizza for lunch or I'll have a salad for lunch. **$p \vee s$**
P_2: I'm not having pizza for lunch. **$\sim p$**

Therefore, I'll have a salad for lunch. **s** **Law of Disjunctive Inference**

P_1: If I clean my room, then I can go to the movies. **$c \rightarrow m$**
P_2: If I go to the movies, then my friends will meet me at the mall. **$m \rightarrow f$**

Therefore, If I clean my room, then my friends will meet me at the mall. **$c \rightarrow f$**
Law of Syllogism

P_1: Conditional statement: If I stay up late, then I will be tired in the morning. **$l \rightarrow t$**

Therefore,
Contrapositive statement: If I am <u>not</u> tired in the morning, then I did <u>not</u> stay up late.
$\sim t \rightarrow \sim l$
Law of Contrapositive Equivalence

<u>State</u> a law of logic listed above that can be used to draw a valid <u>conclusion</u>. Follow the <u>pattern</u> of the rules.

EXAMPLE: P_1: ~k $\rightarrow$ ~h
P_2: h

Conclusion: k

Law: Contrapostive Inference (<u>opposite</u> of the conclusion (~h) implies the <u>opposite</u> of the hypothesis (~k).)

42. P_1: ~f $\rightarrow$ g
P_2: ~f

Conclusion: ____________________________ Law: ________________________________

43. P_1: d $\rightarrow$ w
P_2: ~w

Conclusion: ____________________________ Law: ________________________________

44. P_1: r $\rightarrow$ ~g
P_2: g

Conclusion: ____________________________ Law: ________________________________

45. P_1: h $\vee$ d
P_2: ~d

Conclusion: ____________________________ Law: ________________________________

46. P_1: c $\rightarrow$ ~k
P_2: ~k $\rightarrow$ m

Conclusion: ____________________________ Law: ________________________________

Show that the indicated conclusion follows from the given premises. Use the rules of logic from the previous pages to create a valid argument.

47. P_1: $p \to r$
P_2: $\sim p \to d$
Therefore, C: $\sim d \to r$

Steps	Reasons
1. $\sim p \to d$	1. Premise 2
2. $\sim d \to p$	2. ______________________________ Which rule lets you go from 1 to 2?
3. $p \to r$	3. Premise 1
4. $\sim d \to r$	4. ______________________________ Which rule combines steps 2 and 3?

48. P_1: $\sim r \to p$
P_2: $\sim p$
P_3: $r \to s$
Therefore, C: s

Steps	Reasons
1. $\sim r \to p$	1. Premise 1
2. $\sim p$	2. Premise 2
3. r	3. ______________________________ Which rule combines steps 1 and 2?
4. $r \to s$	4. Premise 3
5. s	5. ______________________________ Which rule combines steps 3 and 4?

Now you try one. Put the statements in order to make a valid conclusion. Use the laws of logic. If you are using one of the given statements, write "premise #__" as your reason for why it is true. There is more than one way to do it.

49. P_1: $p \rightarrow k$
P_2: $\sim z \rightarrow \sim k$
P_3: $\sim z$
Therefore, C: $\sim p$

Steps	Reasons
1. ____________	1. ____________________________
2. ____________	2. ____________________________
3. ____________	3. ____________________________
4. ____________	4. ____________________________
5. ____________	5. ____________________________

Don't forget….

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