Summer Work Packet for MPH Math Classes

Students going into AP Calculus BC Sept. 2022

Name: _____

This packet is designed to help students stay current with their math skills.

Each math class expects a certain level of number sense, algebra sense and graph sense in order to be successful in the course.

This packet's main purpose is to help you prepare for the upcoming year. If you have any questions, email Mr. Ochs at jochs@mphschool.org

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The TI 84^{+} calculator is good for use in AP Calculus. It does everything you are allowed to use it for on the AP Exam.

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AP CALCULUS BC
There are two parts to this packet. Follow the set of instructions for each part.

Part I -Series and Sequences

Instructions:
Go to https://www.youtube.com/watch?v=Tj89FA-d0f8 and then answer the following questions.

1. What is the formula for the sum of a geometric series?

2. Answer the following given the series $9 - 6 + 4 - \frac{8}{3} + \ldots$
   a. Write a formula for the series

   b. What is the sum of the series for $n = 10$

   c. What is the sum of the series as $n \to \infty$

Part II - Integration

Instructions: Complete questions 1-10 on a separate sheet of paper. The rest you can complete on a separate paper or on this packet. Some answers are given. Show all work.

1. $\int x^e dx$

2. $\int \frac{3x^2-5x+8}{x^2} dx$

3. $\int \frac{3x^2}{x^3+1} dx$

4. $\int \frac{8}{\sqrt{12-x^2-4x}} dx$

5. $\int \frac{sin(\sqrt{x})}{\sqrt{x}} dx$

6. $\int \frac{3x^5}{\sqrt{x^3-2}} dx$

7. $\int \frac{12x^2}{2x+1} dx$

8. $\int \frac{x}{1+x^2} dx$

9. $\int \frac{2x}{1+x^4} dx$

10. $\int_{-1}^{4} |x-2| dx$

11. Let f be a differentiable function such that $f(3) = 2.345$ and $f'(x) = ln(x^2 + 1)$. What is the value of $f(5)$? Calculator permitted.

12. Which of the following definite integrals are equal to $\lim_{n\to\infty} \sum_{k=1}^{n} \left(-1 + \frac{5k}{n}\right)\frac{5}{n}$

I. $\int_{-1}^{4} \sin x dx$

II. $\int_{0}^{5} \sin(-1 + x)\, dx$

III. $5\int_{0}^{1} \sin(-1 + 5x) dx$

a. I only
b. II only
c. III only
d. I, II, III

13. Which of the following definite integrals is equal to

$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{10k}{n}\left(\sqrt{1+\frac{5k}{n}}\right)\frac{5}{n}$?

a. $\int_1^6 10\sqrt{x}dx$

b. $\int_1^6 2x\sqrt{x}dx$

c. $\int_0^5 10\sqrt{1+x}dx$

d. $\int_0^5 2x\sqrt{1+x}dx$

14. Which of the following is a left Reimann sum approximation of $\int cos(x^2)dx$ from $2 \le x \le 8$ with n subintervals of equal length?

a. $\sum_{k=1}^{n}\left(\cos\left(2+\frac{k-1}{n}\right)^2\right)\frac{1}{n}$

b. $\sum_{k=1}^{n}\left(\cos\left(\frac{6k}{n}\right)^2\right)\frac{6}{n}$

c. $\sum_{k=1}^{n}\left(\cos\left(2+\frac{6(k-1)}{n}\right)^2\right)\frac{6}{n}$

d. $\sum_{k=1}^{n}\left(\cos\left(2+\frac{6k}{n}\right)^2\right)\frac{6}{n}$

Let f be the function given by $f(x) = \int_6^x (-t^2 - t + 6)\, dt$. Where is the function increasing?

15. The intensity of radiation at a distance of x meters from a source is modeled by the function R given by $R(x) = \frac{k}{x^2}$, where k is a positive constant. What is the average intensity of radiation between 10 meters and 50 meters from the source?

16. The average value of a function f over the interval $[-2,3]$ is -6, and the average value of f over the interval $[3,5]$ is 20. What is the average value of f over the interval $[-2,5]$?

17. Let R be the region in the first quadrant bounded above the graph $y = \frac{7}{3}x + 1$ and bounded below by the graph $y = 2^x$ for $0 \leq x \leq 3$. Which of the following definite integrals gives the area of region R?

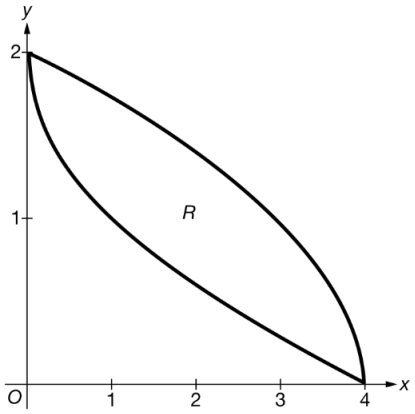
I. $\int_0^3 \left(\left(\frac{7}{3}x + 1\right) - 2^x\right) dx$

II. $\int_0^3 \left(\frac{\ln y}{\ln 2} - \frac{3}{7}\left(y - 1\right)\right) dy$

III. $\int_1^8 \left(\frac{\ln y}{\ln 2} - \frac{3}{7}\left(y - 1\right)\right) dy$

a. None
b. I only
c. I and II only
d. I and III only

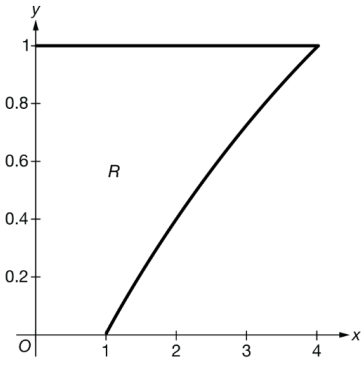
18. Let R be the region in the first quadrant bounded above by the graph of $y = \frac{4}{\pi} arccos(\frac{x}{4})$ and below by the graph $y = 2 - \sqrt{x}$as shown in the figure below. What is the area of the region?

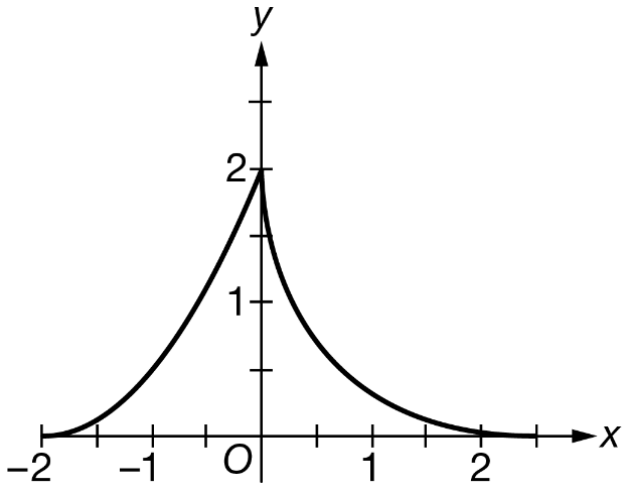


a. $\frac{4}{3}$
b. $\frac{\pi}{16} + \frac{8}{3}$
c. $\frac{\pi}{16} - \frac{8}{3}$
d. $\frac{\pi}{8} + \frac{4}{3}$

19. Let R be the region in the first quadrant bounded by the x- and y-axes, the horizontal line $y = 1$, and the graph of $y = \sqrt{x} - 1$, as shown below. What is the volume of the solid generated when the region R is resolved about the y-axis?

Answer: $\frac{31\pi}{5}$





20. Let f be the function defined by $f(x) = \begin{cases} \frac{1}{2}(x+2)^2 & \text{for } -2 \le x < 0 \\ 2 - 2\sin\sqrt{x} & \text{for } 0 \le x \le \frac{\pi^2}{4} \end{cases}$

The graph of the function is shown above. Let R be the region bounded by the graph of f and the x-axis. Calculator permitted.

a. Find the area of R

b. Region R is the base of a solid. For this solid, each cross-section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

c. The portion of the region R for $1 \leq y \leq 2$ is revolved around the x-axis. Find the volume of the solid.