Summer Enrichment Packet

for

Students going into

Geometry C-Path

Sept. 2023

This summer packet is intended to help students retain the mathematical skills and knowledge they have acquired during the school year, preventing the loss of academic progress.

While completion of this packet is not mandatory, we strongly recommend students utilize this packet. Practicing skills reinforces students' understanding of concepts that they may have struggled with during the school year, helping them to start the new year with a stronger foundation. It can also help students prepare for the challenges of the upcoming school year, and promote problem-solving skills, logical reasoning, and critical thinking abilities, which are valuable not just in math, but in many other areas of life.

****

You will need a TI-84^{+} calculator for this class.

****

Supplies:

TI-84+ calculator
Notebook/binder/paper
Compass (Ones with a wheel, or screw to tighten tend to work better and last longer.)
Protractor
Colored pencils

1. **The Properties of Equality:**

If a = b, then ... a + c = b + c a – c = b – c a × c = b × c a/c = b/c.

These properties are used algebraically for solving equations. In Geometry, they will be used with segments, angles and arcs.

2. **Substitution Property:** If a = c and b = c, then a = b.

This property is used algebraically to evaluate expressions and **<u>check</u>** equations. In Geometry, it will be used to prove segments, angles and arcs equal.

3. **Distributive Property:** a(b + c) = ab + ac and ab + ac = a(b + c) (factoring out "a")

This property is used algebraically to simplify expressions and combine like terms, or for example, to factor a quadratic in order to solve for x. In Geometry, it will also be used to solve equations.

4. **Radicals** (square roots) and quadratic expressions are likely to show up when solving a problem. In Geometry, these are common in problems that deal with Similar and Right Triangles, especially with the Geometric Mean and the Pythagorean Theorem.

$a\sqrt{b}\cdot c\sqrt{d} = ac\sqrt{bd}$ $\qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{a/b}$ $\qquad \sqrt{a}\cdot\sqrt{a} = \sqrt{a^2} = |a|$

$a\sqrt{b} + c\sqrt{b} = (a+c)\sqrt{b}$

Remember, proper form for radical expressions means:

a. No perfect square factor under the radical. For example, $\sqrt{45} = \sqrt{9}\cdot\sqrt{5} = 3\sqrt{5}$

b. No fractions or decimals may be left under the radical. For example, $\sqrt{3/4} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$

c. No radical may be left in the denominator of a fraction. For example,
$\sqrt{3/7} = \frac{\sqrt{3}}{\sqrt{7}} = \frac{\sqrt{3}}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{21}}{7}$. Another example, $\frac{15\sqrt{75}}{20\sqrt{21}} = \frac{3\sqrt{25}\sqrt{3}}{4\sqrt{7}\sqrt{3}} = \frac{3\cdot 5}{4\sqrt{7}} = \frac{15\sqrt{7}}{28}$.

5. It is important to be able to **translate words into mathematics**. This allows you to take information describing the relationship between shapes and turn it into mathematical symbols to solve the problem. These situations can come in many different styles. In algebra, they tend to be referred to as "word problems." In Geometry, the information relates to the various shapes and their specific characteristics.

6. Though we won't specifically use **Logic** notation and all its laws, when we study Geometry we will be proving many theorems. To do this we will set up a series of steps, each of which will be supported by a theorem, definition or postulate. This is similar to creating logic arguments and determining if the stated conclusion is valid and supported by a Law of Reasoning. These Laws of Reasoning are listed with the problems on logic. You will need to follow the examples and check out the symbols.

YOU MAY DO ALL WORK ON THESE PAGES.

NAME _____

Solve each equation. (This is using the Properties of Equality.)
<u>Check</u> your answers. (This is using the Substitution Property.)

1. $\frac{1}{2}x + 5 = \frac{3}{4}x - 7$

2. $4(2x - 5) - 3(x + 2) = -10(x - 3)$

3. $5(3x - 2) = \frac{3}{4}(12x - 16)$

4. $\frac{7x - 4}{6} = \frac{10x + 3}{5}$

Simplify each expression.

5. $\frac{24(-3x^8y^5)}{-12(xy^2)^3}$

6. $(2y - 3x + 4z) - (z - y - 8x) - (3x + 4z - 9y)$

7. $2x + 3(x - 4) - 5(6x - 7)$

8. $\frac{36x^2y}{45xy^4} \cdot (-5x^3y^2)^0$

9. $(a^{-3}b^4c^{-5})(-6a^3b^{-6}c^4)^2$

Use the Distributive Property to write as an expression without parentheses in <u>simplest</u> form.

10. $(x - 6)(x - 7)$

11. $(5x - 6)(x - 9)$

12. $(x + 4)(3x - 8)$

13. $(3x - 5)(3x + 5)$

14. $(4x + 9)(4x + 9)$

15. $(5x + 2)(3x - 10)$

<u>Factor</u> completely, then solve for x algebraically.

16. $x^2 - 11x + 24 = 0$

17. $2x^2 - 7x - 4 = 0$

18. $3x^2 - 10x - 8 = 0$

19. $3x^2 + 10x + 7 = 0$

Solve each of the problems below. Be sure to write the area formula for each.

20. Find the perimeter and area of a square whose side measures $6\sqrt{3}$.

21. Find the area and circumference of a circle with a diameter of 20. Leave π in your answer.

22. The area of a circle is 225π cm^2. Find the length of the diameter.

Simplify the radical expressions. Leave each in its best radical form (no decimal equivalents). Use the notes at the beginning of the packet to help you.

23. $(18 - 2\sqrt{18}) + (12 + 3\sqrt{50})$

24. $\frac{\sqrt{75}}{\sqrt{12}}$

25. $\frac{14\sqrt{72}}{3\sqrt{28}}$

26. $\sqrt{\frac{5}{20}}$

27. $5\sqrt{6} \cdot 3\sqrt{24}$

Write the equation of a line in in <u>slope-intercept</u> form: y = mx + b, where m is the slope and b is the coordinate of the y-intercept.

28. m = 6, (0, –5)

29. m = –½, (8, –6)

30. (–6, –8) and (1, 6)

31. Parallel to the line y = 7x – 5 that goes through the point (–2, –5)

32. Perpendicular to the line 3y – 2x = 12 and goes through the point (–6, 2)

Word Problems

1. A tank sprang a leak, where on average it loses 4 liters of water every 10 hours at a constant rate. If the tank currently has 20 liters of water inside of it, answer the following.
   a. Write a linear equation where L is the amount of water in the tank and h and the amount of hours after the leak started.

   b. Using your equation, how many liters of water will be in the tank after five and a half hours?

   c. Using your equation, how long will it take for the tank to be empty.

2. A towing service requires a flat fee and an additional fee per mile of towing, up to 20 miles. A 4-mile tow costs $110 and a 9-mile tow costs $122.50. Given this information, answer the following.
   a. Write a linear model in slope-intercept form for the price of a tow P(x) for x miles. Give your slope as a decimal in simplest form.

   b. What does the slope represent?

   c. How much is the flat fee? Explain.

   d. Is $155 dollars in the range of your function?

3. A toy company is testing a toy when thrown into the air by launching it from the top of a building. The toy follows the trajectory modeled by $h(t)= -5t2-3t-4$, where t is the seconds after the toy was launched and h(t) is how many feet the toy is from the ground. Given this, algebraically determine the following.

a. How high was the toy off the ground when it was initially launched?

   b. At what time or times does the toy reach the ground?

   c. At what time or times does the toy reach a height of 30 feet?

   d. What is the highest **height** the toy reaches?

LAWS OF LOGIC- For Enhancement- Discussed in class!

$\wedge$ means AND **$\vee$ means OR**

$\rightarrow$ means implies or If..., then... **~ means NOT (opposite)**

p and q are used to represent a simple statement. p: It is sunny outside. q: It is warm outside.

p $\wedge$ q represents "It is sunny outside, **and** it is warm outside."
p $\vee$ q represents "It is sunny outside, **or** it is warm outside."
p $\rightarrow$ q represents "**If** it is sunny outside, **then** it is warm outside."
Or, "It is sunny outside **implies** it is warm outside."
~p represents "It is **not** sunny outside."

If p is true, ~p is false. If p is false, ~p is true. ~(~p) = p

p can be true or false. q can be true of false. There are 4 possible combinations of p and q.

p	q	p $\wedge$ q
T	T	T
T	F	F
F	T	F
F	F	F

"p and q" is TRUE only when BOTH p and q are true.

p	q	p $\vee$ q
T	T	T
T	F	T
F	T	T
F	F	F

"p or q" is FALSE only when BOTH p and q are false.

p	q	p $\rightarrow$ q
T	T	T
T	F	F
F	T	T
F	F	T

"p implies q" is FALSE only when a true statement implies a false one.
(p is the hypothesis and q is the conclusion.)

A conditional statement (p $\rightarrow$ q) has related conditionals statements:
converse: q $\rightarrow$ p (**switch** the hypothesis and conclusion)
inverse: ~p $\rightarrow$ ~q (**negate** the hypothesis and conclusion)
contrapositive: ~q $\rightarrow$ ~p (**switch** and **negate** the hypothesis and conclusion)

Use the information above to do the following.

p: JKLM is a rectangle. q: JKLM is a square.

33. Write $p \wedge q$ in words.

34. Write $p \vee q$ in words.

35. Write $p \rightarrow q$ in words.

36. Write $\sim p$ in words.

37. Explain why "p: JKLM is a rectangle" is **not** the opposite of "q: JKLM is a square.

38. Write the converse, $q \rightarrow p$, in words.

39. Write the inverse, $\sim p \rightarrow \sim q$, in words.

40. Write the contrapositive, $\sim q \rightarrow \sim p$, in words.

41. Which of these 4 conditional statements (#35, 38, 39, 40) are true? Explain.

Laws of Logic (Look for the pattern of the argument.)

A ***premise*****,** represented as P_1, or P_2 etc., is given as true. The ***conclusion*** is represented as **C**.

Law of Detachment: If $(p \rightarrow q)$ is true and p is true, then q is true.
P_1: $p \rightarrow q$
P_2: p
Therefore, C: q

Law of Contrapositive Inference: If $(p \rightarrow q)$ is true and $\sim q$ is true, then $\sim p$ is true.
P_1: $p \rightarrow q$
P_2: $\sim q$
Therefore, C: $\sim p$

Law of Disjunctive Inference: If $(p \vee q)$ is true and $\sim p$ is true, then q is true.

P_1: $p \vee q$	OR	P_1: $p \vee q$
P_2: $\sim p$		P_2: $\sim q$
Therefore, C: q		Therefore, C: p

Law of Syllogism: If $(p \rightarrow q)$ is true and $(q \rightarrow r)$ is true, then $(p \rightarrow r)$ is true.
P_1: $p \rightarrow q$
P_2: $q \rightarrow r$
Therefore, C: $p \rightarrow r$

Law of Contrapositive Equivalence: $p \rightarrow q = \sim q \rightarrow \sim p$
Example: Conditional: If $x + 4 = 7$, then $x = 3$.
Contrapositive: If $x \neq 3$, then $x + 4 \neq 7$.
Both are TRUE.

Examples of how an argument looks and the law that supports it:

P₁: If I study for the test, then I will get an A. **t → a**
P₂: I studied for the test. **t**

Therefore, I will get an A. **a** **Law of Detachment**

P₁: If I study for the test, then I will get an A. **t → a**
P₂: I didn't get an A. **~a**

Therefore, I didn't study for the test. **~t** **Law of Contrapositive Inference**

P₁: I'll have pizza for lunch, or I'll have a salad for lunch. **p ∨ s**
P₂: I'm not having pizza for lunch. **~p**

Therefore, I'll have a salad for lunch. **s** **Law of Disjunctive Inference**

P₁: If I clean my room, then I can go to the movies. **c → m**
P₂: If I go to the movies, then my friends will meet me at the mall. **m → f**

Therefore, If I clean my room, then my friends will meet me at the mall. **c → f**
Law of Syllogism

P₁: Conditional statement: If I stay up late, then I will be tired in the morning. **l → t**

Therefore,
Contrapositive statement: If I am not tired in the morning, then I did not stay up late.
~t → ~l
Law of Contrapositive Equivalence

State a law of logic listed above that can be used to draw a valid conclusion. Follow the pattern of the rules.

EXAMPLE: P_1: $\sim k \rightarrow \sim h$
P_2: h

Conclusion: k Law: Contrapostive Inference (h, the opposite of the conclusion (~h) implies the opposite of the hypothesis (~k).)

42. P_1: $\sim f \rightarrow g$
P_2: $\sim f$

Conclusion: _____ Law: _____

43. P_1: $d \rightarrow w$
P_2: $\sim w$

Conclusion: _____ Law: _____

44. P_1: $r \rightarrow \sim g$
P_2: g

Conclusion: _____ Law: _____

45. P_1: $h \vee d$
P_2: $\sim d$

Conclusion: _____ Law: _____

46. P_1: $c \rightarrow \sim k$
P_2: $\sim k \rightarrow m$

Conclusion: _____ Law: _____

Show that the indicated conclusion follows from the given premises. Use the rules of logic from the previous pages to create a valid argument.

47. P_1: $p \rightarrow r$
P_2: $\sim p \rightarrow d$
Therefore, C: $\sim d \rightarrow r$

Steps	Reasons
1. $\sim p \rightarrow d$	1. Premise 2
2. $\sim d \rightarrow p$	2. _____ Which rule lets you go from 1 to 2?
3. $p \rightarrow r$	3. Premise 1
4. $\sim d \rightarrow r$	4. _____ Which rule combines steps 2 and 3?

48. P_1: $\sim r \rightarrow p$
P_2: $\sim p$
P_3: $r \rightarrow s$
Therefore, C: s

Steps	Reasons
1. $\sim r \rightarrow p$	1. Premise 1
2. $\sim p$	2. Premise 2
3. r	3. _____ Which rule combines steps 1 and 2?
4. $r \rightarrow s$	4. Premise 3
5. s	5. _____ Which rule combines steps 3 and 4?

Now you try one. Put the statements in order to make a valid conclusion. Use the laws of logic. If you are using one of the given statements, write "premise #_" as your reason for why it is true. There is more than one way to do it.

49. P_1: $p \rightarrow k$
P_2: $\sim z \rightarrow \sim k$
P_3: $\sim z$
Therefore, C: $\sim p$

Steps	Reasons
1. _____	1. _____
2. _____	2. _____
3. _____	3. _____
4. _____	4. _____
5. _____	5. _____

Don't forget….

THE NUMBER DEVIL by Hans Magnus Enzenberger

Publisher: Holt Paperbacks (May 1, 2000)

ISBN-10: 0805062998 OR **ISBN-13:** 978-0805062991